Universidad Autónoma de Aguascalientes

Centro de Ciencias Básicas

Departamento de Matemáticas y Física

Cálculo Integral

2º "A"

Cuaderno de Ejercicios de Tarea

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Fecha de Entrega: 11 de febrero del 2019

Tarea 1: Antiderivadas

Calcular la antiderivada más general de las siguientes funciones:

1.- $\int 3ay^2\,dy$

$3a\int y^2\,dy$

$3a\left(\frac{1}{2+1}\cdot y^{2+1}\right)+c$

$3a\left(\frac{1}{3}\cdot y^3\right)+c$

$\frac{3ay^3}{3}+c$

ay^3+c

2.- $\int \sqrt{ax}\,dx$

$\int \sqrt{a}\cdot\sqrt{x}\,dx$

$\sqrt{a}\int \sqrt{x}\,dx$

$\sqrt{a}\left(\frac{1}{\frac{1}{2}+1}\cdot x^{1/2+1}\right)+c$

$\sqrt{a}\left(\frac{1}{\frac{3}{2}}\cdot x^{3/2}\right)+c$

$\frac{2\sqrt{ax^3}}{3}+c$

$\frac{2x\sqrt{ax}}{3}+c$

3.- $\int \sqrt[3]{3t}\,dt$

$\int (3t)^{1/3}\,dt$

$\int (3)^{1/3}(t)^{1/3}\,dt$

$3^{1/3}\int t^{1/3}\,dt$

$3^{1/3}\left(\frac{1}{\frac{1}{3}+1}\cdot t^{1/3+1}\right)+c$

$3^{1/3}\left(\frac{1}{\frac{4}{3}}\cdot t^{4/3}\right)+c$

$3^{1/3}\left(\frac{3}{4}\cdot t^{4/3}\right)+c$

$\frac{\sqrt[3]{3}\cdot 3\cdot t^{4/3}}{4}+c$

$\frac{3t\sqrt[3]{3t}}{4}+c$

4.- $\int \frac{4z^2 - 2\sqrt{z}}{z} \, dz$

$\int \frac{4z^2}{z} - \frac{2\sqrt{z}}{z} \, dz$

$\int 4z - \frac{2}{\sqrt{z}} \, dz$

$\int 4z \, dz - \int \frac{2}{\sqrt{z}} \, dz$

$\left[4\left(\frac{1}{1+1} \cdot z^{1+1}\right) + c_1\right] - \left[2\left(\frac{1}{-\frac{1}{2}+1} \cdot z^{-1/2+1}\right) + c_2\right]$

$4\left(\frac{1}{2} \cdot z^2\right) + c_1 - 2\left(\frac{2}{1} \cdot z^{1/2}\right) + c_2$

$2z^2 - 4\sqrt{z} + c$

5.- $\int \frac{y^3 - 6y + 5}{y} \, dy$

$\int \frac{y^3}{y} - \frac{6y}{y} + \frac{5}{y} \, dy$

$\int y^2 \, dy - \int 6 \, dy + \int \frac{5}{y} \, dy$

$\left(\frac{1}{2+1} \cdot y^{2+1} + c_1\right) - 6\left(\frac{1}{0+1} \cdot y^{0+1}\right) + c^2 + 5 \cdot \ln|y| + c_3$

$\frac{y^3}{3} - 6y + 5\ln|y| + c$

6.- $\int \frac{t+2}{t+1} \, dt$

$\int \frac{t+1+1}{t+1} \, dt$

$\int 1 \, dt + \int \frac{1}{t+1} \, dt$

$1\left(\frac{1}{0+1} \cdot t^{0+1}\right) + c_1 + \ln|t+1| + c_2$

$t + \ln|t+1| + c$

7.- $\int \frac{x^3 + 5x^2 - 4}{x^2}\, dx$

$\int \frac{x^3}{x^2} + \frac{5x^2}{x^2} - \frac{4}{x^2}\, dx$

$\int x\, dx + \int 5\, dx - \int \frac{4}{x^2}\, dx$

$\left(\frac{1}{1+1} \cdot x^{1+1}\right) + c_1 + 5\left(\frac{1}{0+1} \cdot x^{0+1}\right) + c_2 - 4\left(\frac{1}{-2+1} \cdot x^{-2+1}\right) + c_3$

$\frac{x^2}{2} + 5x - \frac{4}{-1} \cdot x^{-1} + c$

$\frac{x^2}{2} + 5x + \frac{4}{x} + c$

8.- $\int (z\sqrt{z} - 2\sqrt[3]{z^2} + 5\sqrt{z} - 3)\, dz$

$\int \left(z^{\frac{3}{2}} - 2z^{\frac{2}{3}} + 5z^{\frac{1}{2}} - 3\right) dz$

$\left(\frac{1}{\frac{3}{2}+1} \cdot z^{3/2+1}\right) - 2\left(\frac{1}{\frac{2}{3}+1} \cdot z^{2/3+1}\right) + 5\left(\frac{1}{\frac{1}{2}+1} \cdot z^{1/2+1}\right) - 3\left(\frac{1}{0+1} \cdot z^{0+1}\right) + c$

$\frac{2}{5} \cdot z^{5/2} - \frac{6}{5} \cdot z^{5/3} + \frac{10}{3} \cdot z^{3/2} - 3z + c$

$\frac{2z^2\sqrt{z}}{5} - \frac{6z\sqrt[3]{z^2}}{5} + \frac{10z\sqrt{z}}{3} - 3z + c$

9.- $\int \left(\frac{y^2}{2} - \frac{2}{y^2}\right) dy$

$\frac{1}{2}\int y^2\, dy - 2\int \frac{1}{y^2}\, dy$

$\frac{1}{2}\left(\frac{1}{2+1} \cdot y^{2+1}\right) - 2\left(\frac{1}{-2+1} \cdot y^{-2+1}\right) + c$

$\frac{1}{2}\left(\frac{1}{3} y^3\right) - 2\left(\frac{1}{-1} \cdot y^{-1}\right) + c$

$\frac{y^3}{6} + \frac{2}{y} + c$

10 = $\int \sqrt{t}\,(3t-2)\,dt$

$$\int 3t\sqrt{t} - 2\sqrt{t}\,dt$$

$$3\int t\sqrt{t}\,dt - 2\int \sqrt{t}$$

$$3\left(\frac{1}{\frac{3}{2}+1}\cdot t^{3/2+1}\right) - 2\left(\frac{1}{\frac{1}{2}+1}\cdot t^{1/2+1}\right) + c$$

$$3\left(\frac{2}{5}\cdot t^{5/2}\right) - 2\left(\frac{2}{3}\cdot t^{3/2}\right) + c$$

$$\frac{6t^2\sqrt{t}}{5} - \frac{4t\sqrt{t}}{3} + c$$

Fecha de Entrega: 18 de Febrero del 2019

Tarea 2: Integración por Sustitución

Calcular la antiderivada más general de las siguientes funciones

1.- $\int (a+bt)^2 dt$

$u = a+bt \qquad du = b\,dt \therefore \frac{du}{b} = dt$

$\int u^2 \left(\frac{du}{b}\right)$

$\frac{1}{b}\int u^2 du$

$\frac{1}{b}\left(\frac{1}{2+1} \cdot u^{2+1}\right) + c$

$\frac{1}{b}\left(\frac{1}{3} \cdot u^3\right) + c$

$\frac{(a+bt)^3}{3b} + c$

2.- $\int x(2+x^2)^2 dx$

$u = 2+x^2 \qquad du = 2x\,dx \therefore \frac{du}{2} = x\,dx$

$\int u^2 \left(\frac{du}{2}\right)$

$\frac{1}{2}\int u^2 du$

$\frac{1}{2}\left(\frac{1}{2+1} \cdot u^{2+1}\right) + c$

$\frac{1}{2}\left(\frac{1}{3} \cdot u^3\right) + c$

$\frac{(2+x^2)^3}{6} + c$

3.- $\int t\sqrt{2t^2+3}\,dt$

$u = 2t^2+3 \qquad du = 4t\,dx \therefore \frac{du}{4} = t\,dt$

$\frac{1}{4}\int u^{1/2} du$

$\frac{1}{4}\left(\frac{1}{\frac{1}{2}+1} u^{1/2+1}\right) + c$

$\frac{1}{4}\left(\frac{2}{3} u^{3/2}\right) + c$

$\frac{(2t^2+3)\sqrt{2t^2+3}}{6} + c$

4.- $\int \frac{4z^2}{\sqrt{z^3+8}}\,dz$

$u = z^3 + 8 \quad du = 3z^2\,dz \quad \therefore \quad \frac{du}{3} = z^2\,dz$

$4\int u^{-1/2}\left(\frac{du}{3}\right)$

$\frac{4}{3}\int u^{-1/2}\,du$

$\frac{4}{3}\left(\frac{1}{-1/2+1}\cdot u^{-1/2+1}\right)+c$

$\frac{4}{3}\left(\frac{2}{1}\cdot u^{1/2}\right)+c$

$\frac{8\sqrt{z^3+8}}{3}+c$

5.- $\int \frac{(\sqrt{a}-\sqrt{y})^2}{\sqrt{y}}\,dy$

$\int \frac{a-2\sqrt{ay}+y}{\sqrt{y}}\,dy$

$\int \frac{a}{\sqrt{y}}\,dy - \int \frac{2\sqrt{ay}}{\sqrt{y}}\,dy + \int \frac{y}{\sqrt{y}}\,dy$

$a\int \frac{1}{\sqrt{y}}\,dy - 2\sqrt{a}\int \frac{\cancel{\sqrt{y}}}{\cancel{\sqrt{y}}}\,dy + \int \frac{y}{\sqrt{y}}\,dy$

$u = y \quad \therefore \quad du = dy$

$a\int u^{-1/2}\,du - 2\sqrt{a}\int 1\,dy + \int u^{1/2}\,du$

$a\left(\frac{1}{-1/2+1}u^{-1/2+1}\right)+c_1 - 2\sqrt{a}\left(\frac{1}{0+1}y^{0+1}\right)+c_2+\left(\frac{1}{1/2+1}u^{1/2+1}\right)+c_3$

$2a\sqrt{y} - 2y\sqrt{a} + \frac{2y\sqrt{y}}{3} + c$

6.- $\int \frac{x}{(a+bx^2)^3}\,dx$

$u = a + bx^2 \qquad du = 2bx\,dx$

$\int u^{-3}\left(\frac{du}{2b}\right)$

$\frac{1}{2b}\left(\frac{1}{-3+1}u^{-3+1}\right)+c$

$\frac{1}{2b}\left(\frac{1}{-2}u^{-2}\right)+c$

$-\frac{1}{4b(a+bx^2)^2}+c$

7 - $\int x^{n-1}\sqrt{a+bx^n}$

$u = a + bx^n$ $\qquad du = nbx^{n-1}\,dx$

$\int u^{1/2}\left(\frac{du}{bn}\right)$

$\frac{1}{bn}\int u^{1/2}\,du$

$\frac{1}{bn}\left(\frac{1}{1/2+1}\cdot u^{1/2+1}\right)+c$

$\frac{2(a+bx^n)\sqrt{a+bx^n}}{bn}+c$

8 - $\int \frac{z+\frac{3}{2}}{\sqrt{z^2+3z}}$

$u = z^2+3z$ $\qquad du = 2z+3\,dz$

$du = 2\left(z+\frac{3}{2}\right)dz$

$\frac{du}{2} = z+\frac{3}{2}\,dz$

$\int u^{-1/2}\left(\frac{du}{2}\right)$

$\frac{1}{2}\int u^{-1/2}\,du$

$\frac{1}{2}\left(\frac{1}{-\frac{1}{2}+1}\cdot u^{-1/2+1}\right)+c$

$\frac{1}{2}\left(\frac{2}{1}\cdot u^{1/2}\right)+c$

$\sqrt{z^2+3z}+c$

9 - $\int \frac{[2+\ln(y)]}{y}\,dy$

$u = 2+\ln(y)$ $\qquad du = \frac{dy}{y}$

$\int u^1\,du$

$\left(\frac{1}{1+1}u^{1+1}\right)+c$

$\frac{u^2}{2}+c$

$\frac{(2+\ln(y))^2}{2}+c$

10.- $\int \frac{1}{t(\ln t)^2}\,dt$

$u = \ln t \qquad du = \frac{dt}{t}$

$\int \frac{du}{u^2}$

$\left(\frac{1}{-2+1} u^{-2+1}\right) + c$

$\left(-\frac{1}{1} u^{-1}\right) + c$

$-\frac{1}{\ln t} + c$

11.- $\int \frac{(x+3)^4}{x+2}\,dx$

$\int \frac{x^4+12x^3+54x^2+108x+81}{x+2}\,dx$

$\int \frac{x^4+12x^3}{x+2} + \frac{54x^2+108x}{x+2} + \frac{81}{x+2}\,dx$

$\int \frac{x^4+12x^3-10x^3+10x^3}{x+2} + \frac{54x^2+108x}{x+2} + \frac{81}{x+2}\,dx$

$\int \frac{x^4+2x^3+10x^3}{x+2} + \frac{54x^2+108x}{x+2} + \frac{81}{x+2}\,dx$

$\int \frac{x^3(x+2)}{x+2} + \frac{10x^3}{x+2} + \frac{54x^2+108x}{x+2} + \frac{81}{x+2}\,dx$

$\int x^3 + \frac{10x^3+20x^2-20x^2}{x+2} + \frac{54x^2+108x}{x+2} + \frac{81}{x+2}\,dx$

$\int x^3 + \frac{10x^2(x+2)}{x+2} - \frac{20x^2}{x+2} + \frac{54x^2+108x}{x+2} + \frac{81}{x+2}\,dx$

$\int x^3+10x^2 - \frac{20x(x+2)}{x+2} - \frac{40x}{x+2} + \frac{54x^2+108x}{x+2} + \frac{81}{x+2}\,dx$

$\int x^3+10x^2-20x-40 - \frac{80}{x+2} + \frac{54x(x+2)}{x+2} + \frac{81}{x+2}\,dx$

$\int x^3+10x^2+34x-40+\frac{1}{x+2}\,dx$

$\int x^3dx + 10\int x^2dx + 34\int x\,dx - \int 40\,dx + \int \frac{dx}{x+2}$

$\left(\frac{1}{3+1}x^{3+1}\right) + 10\left(\frac{1}{2+1}x^{2+1}\right) + 34\left(\frac{1}{1+1}x^{1+1}\right) - 40x + \ln|x+2| + c$

$\frac{x^4}{4} + \frac{10x^3}{3} + 17x^2 - 40x + \ln|x+2| + c$

12- $\int \frac{(y-9)^5}{y-2}\, dy$

$$\int \frac{y^5 - 45y^4 + 810y^3 - 7290y^2 + 32805y - 59049}{y-2}\, dy$$

$$\begin{array}{r l}
 & y^4 - 43y^3 + 724y^2 - 5842y + 21121 \\
y-2 \,\big) & y^5 - 45y^4 + 810y^3 - 7290y^2 + 32805y - 59049 \\
 & \cancel{-y^5} + 2y^4 \\
 & \cancel{-43y^4} + 810y^3 \\
 & \cancel{43y^4} - 86y^3 \\
 & \cancel{724y^3} - 7290y^2 \\
 & \cancel{-724y^3} + 1448y^2 \\
 & \cancel{-5842y^2} + 32805y \\
 & \cancel{5842y^2} - 11684y \\
 & \cancel{21121y} - 59049 \\
 & \cancel{-21121y} + 42242 \\
 & -16807
\end{array}
\qquad \frac{-16807}{y-2}$$

$$\int y^4 - 43y^3 + 724y^2 - 5842y + 21{,}121 - \frac{16807}{y-2}$$

$$\int y^4 dy - 43\int y^3 dy + 724\int y^2 dy - 5842\int y\, dy + \int 21{,}121\, dy - 16807\int \frac{dy}{y-2}$$

$$\frac{y^5}{5} - \frac{43y^4}{4} + \frac{724y^3}{3} - 2921y^2 + 21{,}121y - 16{,}807 \ln|u| + c$$

$$\frac{y^5}{5} - \frac{43y^4}{4} + \frac{724y^3}{3} - 2921y^2 + 21{,}121y - 16807 \ln|y-2| + c$$

Fecha de Entrega: 25 de febrero del 2019

Tarea 3: Integrales Trigonométricas

Calcular la antiderivada más general de las siguientes funciones:

1- $\int y \cot(y^2)\, dy$

$u = y^2 \quad \therefore \quad du = 2y\,dy, \quad \frac{du}{2} = y\,dy$

$\int \cot u \left(\frac{du}{2}\right)$

$\frac{1}{2}\int \cot(u)\, du$

$\frac{1}{2}(\ln|\operatorname{sen} u|) + c$

$\frac{\ln|\operatorname{sen} u|}{2} + c$

$\frac{\ln|\operatorname{sen}(y^2)|}{2} + c$

2- $\int \operatorname{sen}(az)\cos(az)\, dz$

Bajo la identidad: $\operatorname{sen} 2\theta = 2\operatorname{sen}\theta\cos\theta$, sea $\theta = az$

$1 \cdot \int \operatorname{sen}(az)\cos(az)\, dz$

$\frac{2}{2}\int \operatorname{sen}(az)\cos(az)\, dz$

$\frac{1}{2}\int 2\operatorname{sen}(az)\cos(az)\, dz$

$\frac{1}{2}\int \operatorname{sen}(2az)\, dz$

$u = 2az \qquad du = 2a\, dz \qquad \frac{du}{2a} = dz$

$\frac{1}{2}\int \operatorname{sen}(u)\left(\frac{du}{2a}\right)$

$\frac{1}{4a}\int \operatorname{sen}(u)\, du$

$-\frac{\cos(u)}{4a} + c$

$-\frac{\cos(2az)}{4a} + c$

3.- $\int b \sec(ax) \tan(ax)\, dx$

$b \int \sec(ax) \tan(ax)\, dx$

$u = ax \qquad du = a\, dx \quad \therefore \quad \frac{du}{a} = dx$

$b \int \sec(u) \tan(u) \left(\frac{du}{a}\right)$

$\frac{b}{a} \int \sec(u) \tan(u)\, du$

$\frac{b}{a} \cdot \sec u + c$

$\frac{b \sec(ax)}{a} + c$

4.- $\int \sec(\sqrt{y}) \frac{dy}{\sqrt{y}}$

$u = \sqrt{y} \qquad du = \frac{dy}{2\sqrt{y}} \quad \therefore \quad 2\, du = \frac{dy}{\sqrt{y}}$

$\int \sec(u)\, (2\, du)$

$2 \int \sec(u)\, du$

$2 (\ln | \sec u + \tan u |) + c$

$2 (\ln | \sec(\sqrt{y}) + \tan(\sqrt{y}) |) + c$

5.- $\int \frac{1}{2} [\csc^2(x) - \csc(x)\cot(x)]\, dx$

$\int \frac{\csc^2(x)}{2}\, dx - \int \frac{\csc(x)\cot(x)}{2}\, dx$

$\frac{1}{2}[-\cot(x)] - \frac{1}{2}[-\csc(x)] + c$

$\frac{-\cot(x) + \csc(x)}{2} + c$

$\frac{\csc(x) - \cot(x)}{2} + c$

6.- $\int \frac{1 - \cos(6t)}{2}\, dt$

$\int \frac{1}{2}\, dt - \int \frac{\cos(6t)}{2}\, dt$

$u = 6t \qquad du = 6\, dt \qquad \frac{du}{6} = dt$

$\int \frac{dt}{2} - \frac{1}{12} \int \cos(u)\, du$

$\frac{t}{2} - \frac{\operatorname{sen}(u)}{12} + c$

$$\frac{t}{2}-\frac{\operatorname{sen}(6t)}{12}+c$$

$$\frac{6t-\operatorname{sen}(6t)}{12}+c$$

7- $$\int \frac{\csc(z)}{\csc(z)-\operatorname{sen}(z)}\,dz$$

$$\int \csc(z)\left[\frac{1}{\csc(z)-\operatorname{sen}(z)}\right]dz$$

$$\int \csc(z)\left[\frac{1}{\frac{1}{\operatorname{sen}(z)}-\operatorname{sen}(z)}\right]dz$$

$$\int \csc(z)\left[\frac{1}{\frac{1}{\operatorname{sen}(z)}-\frac{\operatorname{sen}^2(z)}{\operatorname{sen}(z)}}\right]dz$$

$$\int \csc(z)\left[\frac{1}{\frac{1-\operatorname{sen}^2(z)}{\operatorname{sen}(z)}}\right]dz$$

$$\int \csc(z)\left[\frac{\operatorname{sen}(z)}{1-\operatorname{sen}^2(z)}\right]dz$$

$$\int \csc(z)\left[\frac{\operatorname{sen}(z)}{\cos^2(z)}\right]dz$$

$$\int \frac{1}{\cancel{\operatorname{sen}(z)}}\cdot\frac{\cancel{\operatorname{sen}(z)}}{\cos^2(z)}\,dz$$

$$\int \frac{1}{\cos^2(z)}\,dz$$

$$\int \sec^2(z)\,dz$$

$$\tan(z)+c$$

8- $$\int \frac{dy}{1-\operatorname{sen}\left(\frac{1}{2}x\right)}$$

$$\frac{y}{1-\operatorname{sen}\left(\frac{x}{2}\right)}$$

9- $$\int \frac{dx}{1+\sec(ax)}$$

$$\int \frac{1}{1+\frac{1}{\cos(ax)}}\,dx$$

Por tangente del ángulo medio: $\cos\theta = \dfrac{1-\tan^2\left(\frac{\theta}{2}\right)}{1+\tan^2\left(\frac{\theta}{2}\right)}$

$\frac{\frac{sen^2}{cos^2}}{\frac{sen}{1}}$

sea $u = ax$ $\qquad du = a\,dx \quad \therefore \quad \frac{du}{a} = dx$

$$\int \frac{1}{1 + \frac{1 + \tan^2\left(\frac{u}{2}\right)}{1 - \tan^2\left(\frac{u}{2}\right)}} \left(\frac{du}{a}\right)$$

$$\frac{1}{a}\int \frac{1}{\frac{\left[1 - \tan^2\left(\frac{u}{2}\right)\right] + \left[1 + \tan^2\left(\frac{u}{2}\right)\right]}{1 - \tan^2\left(\frac{u}{2}\right)}}\, du$$

$$\frac{1}{a}\int \frac{1 - \tan^2\left(\frac{u}{2}\right)}{\left[1 - \tan^2\left(\frac{u}{2}\right)\right] + \left[1 + \tan^2\left(\frac{u}{2}\right)\right]}\, du$$

$$\frac{1}{a}\int \frac{1 - \tan^2\left(\frac{u}{2}\right)}{2}\, du$$

$$\frac{1}{a}\int \frac{1}{2} - \frac{\tan^2\left(\frac{u}{2}\right)}{2}\, du$$

$$\frac{1}{a}\int \frac{1}{2} - \frac{\sec^2\left(\frac{u}{2}\right) - 1}{2}\, du$$

$$\frac{1}{a}\int \frac{1}{2} - \left[\frac{\sec^2\left(\frac{u}{2}\right)}{2} - \frac{1}{2}\right] du$$

$$\frac{1}{a}\int 1 - \frac{\sec^2\left(\frac{u}{2}\right)}{2}\, du$$

$$\frac{1}{a}\left[\int 1\, du - \int \frac{\sec^2\left(\frac{u}{2}\right)}{2}\, du\right]$$

$$\frac{1}{a}\cdot\int a\,dx - \frac{1}{a}\cdot \tan\left(\frac{u}{2}\right)$$

$$x - \frac{\tan\left(\frac{ax}{2}\right)}{a} + c$$

10.- $\displaystyle\int \frac{\sec^3(3t)}{\tan(3t)}\, dt$

$u = 3t \qquad du = 3\,dt \quad \therefore \quad \frac{du}{3} = dt$

$$\int \frac{\sec^3 u}{\tan u}\cdot\frac{du}{3}$$

$$\frac{1}{3}\int \frac{\sec^2 u}{sen\, u}\, du$$

$$\frac{1}{3}\int \frac{\tan^2 u + 1}{sen\, u}\, du$$

$$\frac{1}{3}\int \frac{\tan^2 u}{sen\, u} + \csc u\, du$$

$$\frac{1}{3}\int \tan u \cdot \tan u \cdot \frac{1}{sen\, u} + \csc u\, du$$

$$\frac{1}{3}\int \tan u \cdot \frac{\cancel{sen\, u}}{\cos u}\cdot\frac{1}{\cancel{sen\, u}} + \csc u\, du$$

$$\frac{1}{3}\int \sec u \tan u + \csc u \, du$$
$$\frac{1}{3}\left[\int \sec u \tan u \, du + \int \csc u \, du\right]$$
$$\frac{1}{3}\left[\sec u + \ln|\csc u - \cot u| + c\right]$$
$$\frac{\sec(3t) + \ln|\csc(3t) - \cot(3t)|}{3} + c$$

11.- $\int [\tan(2z) + \sec(2z)]^2 \, dz$

$u = 2z \qquad du = 2\,dz \quad \therefore \quad \frac{du}{2} = dz$

$$\int [\tan u + \sec u]^2 \left(\frac{du}{2}\right)$$
$$\frac{1}{2}\int \tan^2 u + 2\tan u \sec u + \sec^2 u \, du$$
$$\frac{1}{2}\left[\int \tan^2 u \, du + \int 2\tan u \sec u \, du + \int \sec^2 u \, du\right]$$
$$\frac{1}{2}\left[\int \sec^2 u - 1 \, du + 2\int \tan u \sec u \, du + \int \sec^2 u \, du\right]$$
$$\frac{1}{2}\left[\left(\int \sec^2 u \, du\right) - \left(\int 1 \, du\right) + 2\sec u + \tan u + c\right]$$
$$\frac{1}{2}\left[\tan u - u + 2\sec u + \tan u + c\right]$$
$$\frac{1}{2}\left[2\tan u + 2\sec u - u + c\right]$$
$$\tan u + \sec u - \frac{u}{2} + c$$
$$\tan(2z) + \sec(2z) - z + c$$

12.- $\int [\sec(4y) - 1]^2 \, dy$

$u = 4y \qquad du = 4\,dy \quad \therefore \quad \frac{du}{4} = dy$

$$\frac{1}{4}\int [\sec u - 1]^2 \, du$$
$$\frac{1}{4}\int \sec^2 u - 2\sec u + 1 \, du$$
$$\frac{1}{4}\left(\int \sec^2 u \, du - 2\int \sec u \, du + \int 1 \, du\right)$$
$$\frac{1}{4}\left(\tan u - \ln|\sec u + \tan u| + u\right)$$
$$\frac{\tan(4y) - \ln|\sec(4y) + \tan(4y)| + 4y}{4} + c$$

13- $\int \frac{1}{\csc(2t) - \cot(2t)} \, dt$

$u = 2t \quad du = 2\,dt \quad \therefore \frac{du}{2} = dt$

$\frac{1}{2} \int \frac{1}{\csc u - \cot u} \, du$

$\frac{1}{2} \int \frac{1}{\frac{1}{\text{sen}\,u} - \frac{\cos u}{\text{sen}\,u}} \, du$

$\frac{1}{2} \int \frac{\text{sen}\,u}{1 - \cos u} \, du$

$v = 1 - \cos u \quad dv = 0 + \text{sen}\,u \, du$

$\frac{1}{2} \int \frac{dv}{v} = \frac{1}{2} \cdot \ln|v|$

$\frac{\ln|1 - \cos u|}{2} + c$

$\frac{\ln|1 - \cos(2t)|}{2} + c$

Fecha de Entrega: 1° de marzo de 2019

Tarea 4: Sumas y Cálculo de Áreas

1.- Calcular las siguientes sumas

a) $\sum_{j=1}^{6} (3j - 2)$

$\sum_{j=1}^{6} 3j - \sum_{j=1}^{6} 2$

$3\sum_{j=1}^{6} j - 6(2)$

$3\left[\frac{6(6+1)}{2}\right] - 12$

$3(21) - 12$

$63 - 12$

51

b) $\sum_{i=1}^{7} (i+1)^2$

$\sum_{i=1}^{7} (i^2 + 2i + 1)$

$\sum_{i=1}^{7} i^2 + \sum_{i=1}^{7} 2i + \sum_{i=1}^{7} 1$

$\left[\frac{7(8)(15)}{6}\right] + 2\left[\frac{7(8)}{2}\right] + 7$

$140 + 56 + 7$

203

c) $\sum_{k=1}^{10} (k^3 - 1)$

$\sum_{k=1}^{10} k^3 - \sum_{k=1}^{10} 1$

$\left[\frac{10^2(11)^2}{4}\right] - 10$

$3025 - 10$

3015

2.- Aproximar con 4 cifras significativas el area de la región limitada por la curva $y = f(x)$, el eje x y las rectas $x = a$, $x = b$, dividiendo el intervalo $[a, b]$ en n subintervalos

a) $f(x) = \frac{1}{x}$, $a = 1$, $b = 3$, $n = 10$, inscritos

Cada subintervalo medirá $\frac{3-1}{10}\,u = \frac{2}{10}\,u = \frac{1}{5}\,u$

Se tomarán valores de $\frac{6}{5}$ a $\frac{15}{5}$

Para	su altura es	por $\frac{1}{5}$ es
$\frac{6}{5}$	$\frac{5}{6}$	$\frac{1}{6}$
$\frac{7}{5}$	$\frac{5}{7}$	$\frac{1}{7}$
$\frac{8}{5}$	$\frac{5}{8}$	$\frac{1}{8}$
$\frac{9}{5}$	$\frac{5}{9}$	$\frac{1}{9}$
$\frac{10}{5}$	$\frac{5}{10}$	$\frac{1}{10}$
$\frac{11}{5}$	$\frac{5}{11}$	$\frac{1}{11}$
$\frac{12}{5}$	$\frac{5}{12}$	$\frac{1}{12}$
$\frac{13}{5}$	$\frac{5}{13}$	$\frac{1}{13}$
$\frac{14}{5}$	$\frac{5}{14}$	$\frac{1}{14}$
$\frac{15}{5}$	$\frac{5}{15}$	$\frac{1}{15}$

$$\frac{372935}{360360} \rightarrow \frac{74587}{72072} \approx 1.0349$$

b) $f(x) = \text{sen}(x)$, $a = \frac{\pi}{6}$, $b = \frac{5\pi}{6}$, $n = 8$, circunscritos

$$\Delta x = \frac{\frac{5\pi}{6} - \frac{\pi}{6}}{8} = \frac{\frac{2\pi}{3}}{8} = \frac{\pi}{12}$$

Como la función seno tiene un comportamiento ascendente y descendente, para que los rectángulos sean circunscritos, es decir, que la gráfica esté dentro de los rectángulos, se debe dividir la función en $\frac{\pi}{2}$

En $\left[\frac{\pi}{6}, \frac{\pi}{2}\right)$ asciende y en $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right]$ desciende

Análisis de cuando la función asciende:

$x_1 = \frac{\pi}{4}$, $x_2 = \frac{\pi}{3}$, $x_3 = \frac{5\pi}{12}$, $x_4 = \frac{\pi}{2}$

$f(x_1) = \frac{\sqrt{2}}{2}$, $f(x_2) = \frac{\sqrt{3}}{2}$, $f(x_3) = \frac{\sqrt{6} + \sqrt{2}}{4}$, $f(x_4) = 1$

$A(x_1) = \frac{\pi\sqrt{2}}{24}$, $A(x_2) = \frac{\pi\sqrt{3}}{24}$, $A(x_3) = \frac{\pi(\sqrt{6} + \sqrt{2})}{48}$, $A(x_4) = \frac{\pi}{12}$

$A_1 = 0.9265$

Sin embargo, como la función seno es reflexiva si se toma como eje de simetría cualquier punto crítico, en este caso, $\frac{\pi}{2}$ y como los límites inferior y superior se encuentran a la misma distancia del punto crítico, entonces el área de cuando la función desciende hasta $\frac{5\pi}{6}$ será equivalente a cuando asciende desde $\frac{\pi}{6}$

$$\text{Área final} = 2A_1 = 2(0.9265) = 1.8530$$

c) $f(x) = x^2$, $a=1$, $b=2$, $n=8$, circunscritos

$\Delta x = \frac{1}{8}$

$x_i = 1 + i\left(\frac{1}{8}\right)$

El área de cada rectángulo se define por el producto de $f(x_i)\Delta x$

$$\sum_{i=1}^{8}\left[\left(1+\frac{i}{8}\right)^2\left(\frac{1}{8}\right)\right]$$

$$\sum_{i=1}^{8}\left[\left(1+\frac{2i}{8}+\frac{i^2}{64}\right)\left(\frac{1}{8}\right)\right]$$

$$\sum_{i=1}^{8}\left(\frac{1}{8}+\frac{i}{32}+\frac{i^2}{512}\right)$$

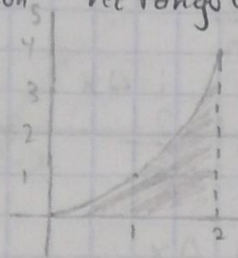
$$\sum_{i=1}^{8}\left(\frac{1}{8}\right)+\sum_{i=1}^{8}\left(\frac{i}{32}\right)+\sum_{i=1}^{8}\left(\frac{i^2}{512}\right)$$

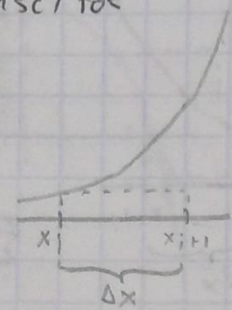
$$1+\frac{1}{32}\left[\frac{8(9)}{2}\right]+\frac{1}{512}\left[\frac{8(9)(17)}{6}\right]$$

$$1+\frac{9}{8}+\frac{51}{128}=\frac{323}{128}=2.5234$$

3: Calcular el área utilizando n rectángulos, para cada ejercicio dibujar la región y el i-ésimo rectángulo

a) La región limitada por $y = x^2$, el eje x, la recta 2 con rectángulos inscritos





$\Delta x = \frac{2}{n}$

$x_i = 0 + (i-1)\left(\frac{2}{n}\right)$

$\sum_{i=1}^{n+1} f(x_i)\,\Delta x$

$$\sum_{i=1}^{n}\left[\left(\frac{2i}{n} - \frac{2}{n}\right)^2\left(\frac{2}{n}\right)\right]$$

$$\sum_{i=1}^{n}\left[\left(\frac{4i^2}{n^2} - \frac{8i}{n^2} + \frac{4}{n^2}\right)\left(\frac{2}{n}\right)\right]$$

$$\sum_{i=1}^{n}\left(\frac{8i^2}{n^3} - \frac{16i}{n^3} + \frac{8}{n^3}\right)$$

$$\frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{16}{n^3}\left(\frac{n(n+1)}{2}\right) + \frac{8}{n^3}$$

$$\frac{8}{6}\left[\frac{n}{n}\cdot\frac{n+1}{n}\cdot\frac{2n+1}{n}\right] - \frac{16}{2n}\left[\frac{n}{n}\cdot\frac{n+1}{n}\right] + \frac{8}{n^3}$$

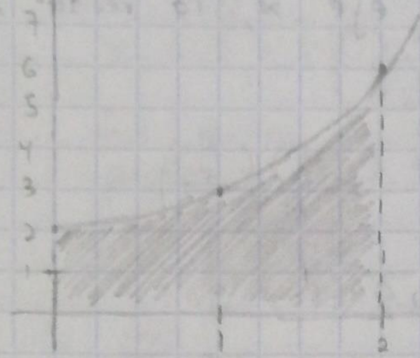
$$\frac{8}{6}\left[\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] - \frac{16}{2n}\left[1+\frac{1}{n}\right] + \frac{8}{n^3}$$

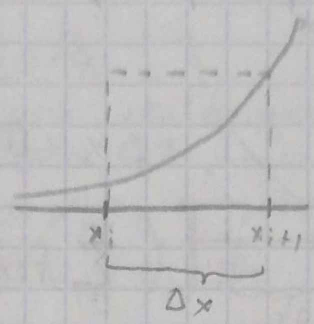
$$\frac{8}{6}\left(\frac{1}{n^2} + \frac{3}{n} + 2\right) - \frac{16}{2n} - \frac{16}{2n^2} + \frac{8}{n^3}$$

$$\frac{8}{6n^2} + \frac{8}{2n} + \frac{8}{3} - \frac{16}{2n} - \frac{16}{2n^2} + \frac{8}{n^3}$$

$$\frac{8}{n^3} - \frac{40}{n^2} - \frac{4}{n} + \frac{8}{3}$$

b) La región limitada por $y = x^2 + 2$, el eje x, las rectas $x = 0$, $x = 2$ con rectángulos circunscritos





$\Delta x = \frac{2}{n}$

$x_i = a + i\,\Delta x$

$x_i = 0 + \frac{2i}{n}$

$\sum_{i=1}^{n} f(x_i)\,\Delta x$

$$\sum_{i=1}^{n} \left\{\left[\left(\frac{2i}{n}\right)^2 + 2\right] \cdot \frac{2}{n}\right\}$$

$$\sum_{i=1}^{n} \left[\left(\frac{4i^2}{n^2} + 2\right) \cdot \frac{2}{n}\right]$$

$$\sum_{i=1}^{n} \left(\frac{8i^2}{n^3} + \frac{4}{n}\right)$$

$$\sum_{i=1}^{n} \frac{8i^2}{n^3} + \sum_{i=1}^{n} \frac{4}{n}$$

$$\frac{8}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{4n}{n}$$

$$\frac{4}{3}\left[\frac{(n+1)(2n+1)}{n^2}\right] + 4$$

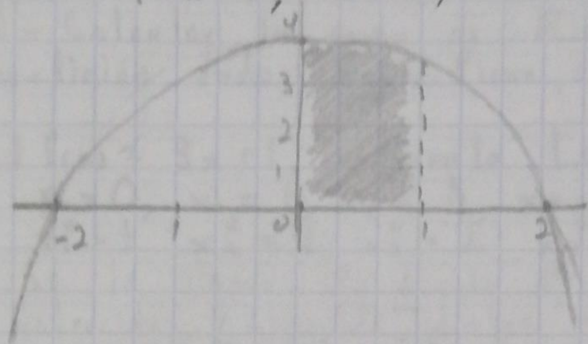
$$\frac{4}{3}\left[\frac{2n^2 + 3n + 1}{n^2}\right] + 4$$

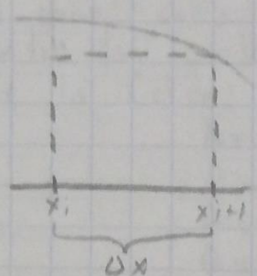
$$\frac{4}{3}\left[2 + \frac{3}{n} + \frac{1}{n^2}\right] + 4$$

$$\frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} + 4$$

$$\frac{20}{3} + \frac{4}{n} + \frac{4}{3n^2}$$

c) La región limitada por $y = 4 - x^2$, el eje x, las rectas $x = 0$, $x = 1$, con rectángulos inscritos





$$\Delta x = \frac{1}{n}$$

$$x_i = 0 + \frac{i}{n}$$

$$\sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$\sum_{i=1}^{n} \left\{ \left[4 - \left(\frac{i}{n}\right)^2 \right] \cdot \frac{1}{n} \right\}$$

$$\sum_{i=1}^{n} \left[\left(4 - \frac{i^2}{n^2}\right) \cdot \frac{1}{n} \right]$$

$$\sum_{i=1}^{n} \left(\frac{4}{n} - \frac{i^2}{n^3} \right)$$

$$\sum_{i=1}^{n} \frac{4}{n} - \sum_{i=1}^{n} \frac{i^2}{n^3}$$

$$4 - \frac{1}{n^3} \left[\frac{n(n+1)(2n+1)}{6} \right]$$

$$4 - \frac{1}{6} \left[\frac{(n+1)(2n+1)}{n^2} \right]$$

$$4 - \frac{1}{6} \left[\frac{2n^2 + 3n + 1}{n^2} \right]$$

$$4 - \frac{1}{6} \left(2 + \frac{3}{n} + \frac{1}{n^2} \right)$$

$$4 - \frac{1}{3} - \frac{1}{2n} - \frac{1}{6n^2}$$

$$\frac{11}{3} - \frac{1}{2n} - \frac{1}{6n^2}$$

Fecha de Entrega: 11 de marzo de 2019

Tarea 5: Sumas de Riemann e Integrales Definidas

1 - Calcular la suma de Riemann $\sum_{k=1}^{n} f(x_k^*)\Delta x_k$ para la partición dada. Especificar $\|\Delta\|$

a) $f(x) = 3x+1$. Intervalo $[0, 3]$. Cuatro subintervalos

$x_0 = 0,\ x_1 = 1,\ x_2 = \frac{5}{3},\ x_3 = \frac{7}{3},\ x_4 = 3$

$x_1^* = \frac{1}{2},\ x_2^* = \frac{4}{3},\ x_3^* = 2,\ x_4^* = \frac{8}{3}$

Se le llama Partición P de un intervalo cerrado a toda colección finita de puntos contenidos en el intervalo

$$P = \{0, 1, \tfrac{5}{3}, \tfrac{7}{3}, 3\}$$

A cada subintervalo se le conoce como Celda. La distancia entre los puntos extremos de cada celda se le llama Amplitud de la Celda:

$$\Delta_1 x = 1 - 0 = 1$$
$$\Delta_2 x = \frac{2}{3}$$
$$\Delta_3 x = \frac{2}{3}$$
$$\Delta_4 x = \frac{2}{3}$$

A la mayor amplitud de las celdas de una partición se le denomina Norma de la Partición, denotada como $\|\Delta\|$

Por lo tanto: $\|\Delta\| = 1$

$$\sum_{k=1}^{4} f(x_k^*)\Delta x_k = \left[3\left(\frac{1}{2}\right)+1\right](1) + \left[3\left(\frac{4}{3}\right)+1\right]\left(\frac{2}{3}\right) + [3(2)+1]\left(\frac{2}{3}\right) + \left[3\left(\frac{8}{3}\right)+1\right]\left(\frac{2}{3}\right)$$

$$= \frac{5}{2}(1) + 5\left(\frac{2}{3}\right) + 7\left(\frac{2}{3}\right) + 9\left(\frac{2}{3}\right)$$

$$= \frac{5}{2} + \frac{10}{3} + \frac{14}{3} + 6$$

$$= \frac{5}{2} + 8 + 6$$

$$= 14 + \frac{5}{2}$$

$$= \frac{33}{2}$$

b) $f(x) = x^2$. Intervalo $[-1, 1]$. Cuatro subintervalos

$x_0 = -1, \quad x_1 = -\frac{1}{4}, \quad x_2 = \frac{1}{4}, \quad x_3 = \frac{3}{4}, \quad x_4 = 1$

$x_1^* = -\frac{3}{4}, \quad x_2^* = 0, \quad x_3^* = \frac{1}{2}, \quad x_4^* = \frac{7}{8}$

$$\left.\begin{array}{l} \Delta_1 x = \frac{3}{4} \\ \Delta_2 x = \frac{1}{2} \\ \Delta_3 x = \frac{1}{2} \\ \Delta_4 x = \frac{1}{4} \end{array}\right\} \therefore \|\Delta\| = \frac{3}{4}$$

$$\sum_{k=1}^{4} f(x_k^*)\Delta x_k = \left(-\frac{3}{4}\right)^2\left(\frac{3}{4}\right) + (0)^2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + \left(\frac{7}{8}\right)^2\left(\frac{1}{4}\right)$$

$$= \frac{27}{64} + 0 + \frac{1}{8} + \frac{49}{256} = \frac{108}{256} + \frac{32}{256} + \frac{49}{256}$$

$$= \frac{189}{256}$$

c) $f(x) = \operatorname{sen}(x)$. Intervalo $[0, 2\pi]$. Tres subintervalos

$x_0 = 0, \quad x_1 = \pi, \quad x_2 = \frac{3\pi}{2}, \quad x_3 = 2\pi$

$x_1^* = \frac{\pi}{2}, \quad x_2^* = \frac{7\pi}{6}, \quad x_3^* = \frac{7\pi}{4}$

$$\left.\begin{array}{l} \Delta_1 x = \pi \\ \Delta_2 x = \frac{\pi}{2} \\ \Delta_3 x = \frac{\pi}{2} \end{array}\right\} \therefore \|\Delta\| = \pi$$

$$\sum_{k=1}^{3} f(x_k^*)\Delta x_k = \operatorname{sen}\left(\frac{\pi}{2}\right)\cdot\pi + \operatorname{sen}\left(\frac{7\pi}{6}\right)\cdot\frac{\pi}{2} + \operatorname{sen}\left(\frac{7\pi}{4}\right)\cdot\frac{\pi}{2}$$

$$= 1\cdot\pi + \left(-\frac{1}{2}\right)\left(\frac{\pi}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\pi}{2}\right)$$

$$= \pi - \frac{\pi}{4} - \frac{\pi\sqrt{2}}{4}$$

$$= \frac{3\pi - \pi\sqrt{2}}{4} \approx 1.2455$$

d) $f(x) = x - 2$. Intervalo $[0, 5]$. Cinco subintervalos de la misma longitud. x_k^* es el punto derecho de cada subintervalo

$\|\Delta\| = 1 \qquad \Delta x = 1 \qquad x_k = k$

$$\sum_{k=1}^{5} (k-2)(1) = \sum_{k=1}^{5} k - \sum_{k=1}^{5} 2$$

$$\frac{5(6)}{2} - 10$$

$$15 - 10$$

$$5$$

2.- Evaluar las siguientes integrales definidas si se sabe que: $\int_a^b dx = b - a$ $\int_a^b x\,dx = \frac{1}{2}(b^2 - a^2)$ $\int_a^b x^2\,dx = \frac{1}{3}(b^3 - a^3)$

a) $-\int_3^{-1} 10x\,dx$

$\int_{-1}^{3} 10x\,dx \rightarrow 10\int_{-1}^{3} x\,dx$

$10\left[\frac{1}{2}(9-1)\right]$

$10(4) = 40$

b) $\int_3^{-1} (-3x^2 + 4x - 5)\,dx$

$-\int_{-1}^{3} (-3x^2 + 4x - 5)\,dx$

$-\int_{-1}^{3} -3x^2\,dx - \int_{-1}^{3} 4x\,dx - \int_{-1}^{3} -5\,dx$

$3\int_{-1}^{3} x^2\,dx - 4\int_{-1}^{3} x\,dx + 5\int_{-1}^{3} dx$

$3\left[\frac{1}{3}(27+1)\right] - 4\left[\frac{1}{2}(9-1)\right] + 5(3+1)$

$28 - 16 + 20$

32

c) $\int_{-1}^{0} x^2\,dx + \int_0^3 x^2\,dx$

$\int_{-1}^{3} x^2\,dx$

$\frac{1}{3}(27+1)$

$\frac{28}{3}$

d) $\int_0^4 x\,dx + \int_0^4 (9-x)\,dx$

$\cancel{\int_0^4 x\,dx} + \int_0^4 9\,dx - \cancel{\int_0^4 x\,dx}$

$\int_0^4 9\,dx$

$9(4-0)$

36

e) $\int_{-1}^{0} t^2\,dt + \int_{0}^{2} x^2\,dx + \int_{2}^{3} y^2\,dy$

$\int_{-1}^{3} z^2\,dz$

$\frac{28}{3}$

f) $\int_{0}^{3} x^2 + \int_{3}^{0} u^2\,du$

$\int_{0}^{3} x^2\,dx - \int_{0}^{3} u^2\,du$

0

g) $\int_{-1}^{-1} 5x - \int_{3}^{-1} (t-4)\,dt$

$\int_{-1}^{3} (t-4)\,dt$

$\int_{-1}^{3} t\,dt - 4\int_{-1}^{3} dt$

$\frac{1}{2}(9-1) - 4(3+1)$

$4 - 16$

-12

3.- Evaluar la integral dada utilizando la información proporcionada

a) $\int_{2}^{5} f(x)\,dx$ si $\int_{0}^{2} f(x)\,dx = 6$ y $\int_{0}^{5} f(x)\,dx = 8.5$

$\int_{2}^{5} f(x)\,dx = 2.5$

b) $\int_{-1}^{2} [2f(x) + g(x)]\,dx$ si $\int_{-1}^{2} f(x)\,dx = 3.4$ y $\int_{-1}^{2} 3g(x)\,dx = 12.6$

$\int_{-1}^{2} [2f(x) + g(x)]\,dx = 2(3.4) + \frac{12.6}{3}$

$= 6.8 + 4.2$

$= 11$

c) $\int_{-2}^{2} g(x)\,dx$ si $\int_{2}^{-2} f(x)\,dx = 14$ y $\int_{-2}^{2} [f(x) - 5g(x)]\,dx = 24$

$-14 - 5z = 24$

$-5z = 38$

$z = -\frac{38}{5}$

$\int_{-2}^{2} g(x)\,dx = -7.6$

Fecha de Entrega: 11 de marzo del 2019

Tarea 6: Teorema Fundamental del Cálculo

1.- Evalúa las siguientes integrales

a) $\int_0^4 (y^3 - y^2 + 1)\, dy$

$\frac{y^4}{4} - \frac{y^3}{3} + y \Big]_0^4$

$\left(64 - \frac{64}{3} + 4\right) - 0$

$68 - \frac{64}{3}$

$\frac{140}{3}$

b) $\int_{-3}^{5} (z^3 - 4z)\, dz$

$\frac{z^4}{4} - 2z^2 \Big]_{-3}^{5}$

$\left(\frac{625}{4} - 50\right) - \left(\frac{81}{4} - 18\right)$

$\frac{544}{4} - 32$

$\frac{416}{4} = 104$

c) $\int_1^4 \sqrt{t}\,(t+2)\, dt$

$\int_1^4 t^{3/2} + 2t^{1/2}\, dt$

$\frac{2t^{5/2}}{5} + \frac{4t^{3/2}}{3} \Big]_1^4$

$\left(\frac{2(4)^{5/2}}{5} + \frac{4(4)^{3/2}}{3}\right) - \left(\frac{2(1)^{5/2}}{5} + \frac{4(1)^{3/2}}{3}\right)$

$\frac{352}{15} - \frac{26}{15}$

$\frac{326}{15}$

d) $\int_0^{\sqrt{5}} v\sqrt{v^2+1}\, dv \qquad u = v^2 + 1 \qquad du = 2v\, dv$

$\frac{1}{2} \cdot \frac{2(v^2+1)^{3/2}}{3} = \frac{(v^2+1)^{3/2}}{3} \Big]_0^{\sqrt{5}}$

$\frac{(5+1)\sqrt{5+1}}{3} - \frac{(1)\sqrt{1}}{3}$

$\frac{6\sqrt{6} - 1}{3} \approx 4.5656$

e) $\int_{-1}^{3} \frac{dy}{(y+2)^2} = -\frac{1}{y+2}\Big]_{-1}^{3}$

$-1 - \left(-\frac{1}{5}\right)$

$-1 + \frac{1}{5} = -\frac{4}{5}$

f) $\int_{0}^{\pi} \cos\left(\frac{x}{2}\right) dx \longrightarrow 2\,\text{sen}\left(\frac{x}{2}\right)\Big]_{0}^{\pi}$

$2\,\text{sen}\left(\frac{\pi}{2}\right) - 2\,\text{sen}\left(\frac{0}{2}\right)$

2

g) $\int_{0}^{1} \frac{t^2 + 2t}{\sqrt[3]{t^3+3t^2+4}}\, dt \longrightarrow \frac{\sqrt[3]{(t^3+3t^2+4)^2}}{2}\Big]_{0}^{1}$

$\frac{\sqrt[3]{(1^3+3\cdot 1^2+4)^2}}{2} - \frac{\sqrt[3]{(0^3+3\cdot 0^2+4)^2}}{2}$

$4 - \frac{\sqrt[3]{16}}{2} \approx 0.7401$

h) $\int_{0}^{15} \frac{z}{(z+1)^{3/4}}\, dz \longrightarrow \frac{4(z+1)^{5/4}}{5} - 4\sqrt[4]{z+1}\Big]_{0}^{15}$

$\frac{4(15-4)\sqrt[4]{15+1}}{5} - \frac{4(-4)\sqrt[4]{1}}{5}$

20.8

i) $\int_{-2}^{5} |x-3|\, dx \quad x-3 \geq 0 \quad x \geq 3$

$-(x-3) < 0 \longrightarrow -x+3$

$\int_{-2}^{3} -x+3\, dx + \int_{3}^{5} x-3\, dx$

$\left(3x - \frac{x^2}{2}\right) + \left(\frac{x^2}{2} - 3x\right)$

$\left[\left(9 - \frac{9}{2}\right) - \left(-6 - 2\right)\right] - \left[\left(\frac{25}{2} - 15\right) - \left(\frac{9}{2} - 9\right)\right]$

$9 - \cancel{\frac{9}{2}} + 6 + 2 - \frac{25}{2} + 15 + \cancel{\frac{9}{2}} - 9$

$23 - \frac{25}{2}$

$23 - 8 - \frac{1}{2}$

14.5

j) $\int_{-1}^{1} \sqrt{|x| - x}\, dx = \int \sqrt{\sqrt{x^2} - x}\, dx$

$\int_{-1}^{0} \sqrt{-x-x}\, dx + \int_0^1 \sqrt{x-x}\, dx$

$\int_{-1}^{0} \sqrt{-2x}\, dx = -\frac{2\sqrt{2} \cdot \sqrt{(-x)^3}}{3}$

$\frac{2\sqrt{2} \cdot \sqrt{(-(-1))^3}}{3} - 0$

0.9428

k) $\int_0^3 (t+2)\sqrt{t+1}\, dt \longrightarrow \frac{2(x+1)^{5/2}}{5} + \frac{2(x+1)^{3/2}}{3} \Big]_0^3$

$\left(\frac{2(3+1)^{5/2}}{5} + \frac{2(3+1)^{3/2}}{3}\right) - \left(\frac{2(1)^{5/2}}{5} + \frac{2(1)^{3/2}}{3}\right)$

17.0667

2- Calcular las siguientes derivadas

a) $\frac{d}{dx} \int_0^x \sqrt{4+t^6}\, dt$

$\sqrt{4+x^6}$

b) $\frac{d}{dx} \int_x^3 \operatorname{sen}(t)\, dt$

$\frac{d}{dx} - \int_3^x \operatorname{sen}(t)\, dt$

$-\operatorname{sen}(x)$

c) $\frac{d}{dx} \int_0^x \frac{du}{u^4+4}$

$\frac{1}{x^4+4}$

d) $\frac{d}{dx} \int_{-x}^{x} \frac{dz}{3+z^2}$

$\frac{d}{dx} \left(\int_{-x}^{0} \frac{dz}{3+z^2} + \int_0^x \frac{dz}{3+z^2} \right)$

$\frac{1}{3+x^2} + \frac{1}{3+x^2} = \frac{2}{3+x^2}$

e) $\frac{d}{dx} \int_{-x}^{x} \cos(u^2+1)\, du$

$\frac{d}{dx} \left(\int_{-x}^{0} \cos(u^2+1) + \int_0^x \cos(u^2+1) \right)$

$2 \cdot \cos(x^2+1)$

f) $\frac{d}{dx}\int_{x}^{x^2}\frac{dy}{\sqrt{y^2+1}}$

$\frac{d}{dx}\left(\int_0^{x^2}\frac{dy}{\sqrt{y^2+1}} - \int_0^{x}\frac{dy}{\sqrt{y^2+1}}\right)$

$\frac{2x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{y^2+1}}$

g) $\frac{d}{dx}\int_2^{\sec(x)}\frac{dv}{v^2-1}$

$\frac{\sec x \tan x}{x^2-1}$

h) $\frac{d}{dx}\int_3^{\operatorname{sen}(x)}\frac{dt}{1-t^2}$

$\frac{\cos(x)}{1-x^2}$

Fecha de Entrega: 28 de marzo del 2019 10

Tarea 7: Integración de funciones trigonométricas inversas

1.- Evalúa las siguientes integrales:

a) $\int \frac{dx}{\sqrt{1-4x^2}}$ $\quad a^2=1 \quad u^2=4x^2$
$a=1 \quad u=2x \quad \therefore du=2dx$

$\int \frac{\frac{du}{2}}{\sqrt{a^2-u^2}}$

$\frac{1}{2}\int \frac{du}{\sqrt{a^2-u^2}}$

$\frac{1}{2}\cdot \operatorname{sen}^{-1}\left(\frac{u}{a}\right)+c$

$\frac{\operatorname{sen}^{-1}(2x)}{2}+c$

OK

b) $\int \frac{dy}{9+3y^2}$ $\quad a^2=9 \quad u^2=3y^2$
$a=3 \quad u=\sqrt{3}\,y \quad \therefore du=\sqrt{3}\,dy$

$\int \frac{\frac{du}{\sqrt{3}}}{a^2+u^2}$

$\frac{1}{\sqrt{3}}\int \frac{du}{a^2+u^2}$

$\frac{1}{\sqrt{3}}\cdot\frac{1}{a}\cdot \tan^{-1}\left(\frac{u}{a}\right)+c$

$\frac{1}{\sqrt{3}}\cdot\frac{1}{3}\cdot \tan^{-1}\left(\frac{\sqrt{3}\,y}{3}\right)+c$

$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\,y}{3}\right)}{9}+c$

OK

c) $\int \frac{dt}{t\sqrt{5t^2-4}}$ $\quad a^2=4 \quad u^2=5t^2$
$a=2 \quad u=\sqrt{5}\,t \quad du=\sqrt{5}\,dt$

$\int \frac{\frac{du}{\sqrt{5}}}{\frac{u}{\sqrt{5}}\sqrt{u^2-a^2}} = \int \frac{du}{u\sqrt{u^2-a^2}}$ OK

$\frac{1}{a}\cdot \sec^{-1}\left(\frac{u}{a}\right)+c$

$\frac{1}{2}\cdot \sec^{-1}\left(\frac{\sqrt{5}\,t}{2}\right)+c$

$\frac{\sec^{-1}\left(\frac{\sqrt{5}\,t}{2}\right)}{2}+c$

d) $\int \frac{dt}{\sqrt{2t-t^2}}$

$\int \frac{dt}{\sqrt{1-(t-1)^2}}$ $\quad a^2 = 1 \quad a = 1 \quad u^2 = (t-1)^2 \quad u = t-1 \quad du = dt$

$\int \frac{du}{\sqrt{a^2-u^2}}$

$\operatorname{sen}^{-1}\left(\frac{u}{a}\right) + c$

$\operatorname{sen}^{-1}\left(\frac{t-1}{1}\right) + c$

$\operatorname{sen}^{-1}(t-1) + c$

OK

e) $\int \frac{z-2}{z^2-6z+10}\, dz$

$\int \frac{z-2}{(z-3)^2+1}$

$z-2 = \frac{1}{2}(2z-6)+1$

$\int \frac{(2z-6)+1}{2[(z-3)^2+1]} = \int \frac{2z-6}{2[z^2-6z+10]} + \int \frac{1}{[(z-3)^2+1]}$

$\frac{1}{2}\int \frac{2z-6}{z^2-6z+10} + \int \frac{1}{(z-3)^2+1}$

OK

Sean: $u = z^2-6z+10 \qquad v = z-3$

$du = 2z-6\, dz \qquad dv = dz$

$\frac{1}{2}\int \frac{du}{u} + \int \frac{1}{v^2+1}\, dv$

$\frac{\ln|u|}{2} + \tan^{-1}(v) + c$

$\frac{\ln|z^2-6z+10|}{2} + \tan^{-1}(z-3) + c$

f) $\int \frac{r^3-2r^2+3r-4}{r^2+1}\, dr$

$$\begin{array}{r|l} & r-2 \\ r^2+1 & r^3-2r^2+3r-4 \\ & -r^3 \quad\quad -r \\ & 0 \; -2r^2+2r \\ & +2r^2 \quad\quad +2 \\ & 0 \quad +2r \; -2 \end{array}$$

OK

$\int \left(r-2+\frac{2r-2}{r^2+1}\right) dr$

$\int r\,dr - 2\int dr + \int \frac{2r-2}{r^2+1}\, dr$

$\int r\,dr - 2\int dr + 2\left(\int \frac{r}{r^2+1} - \frac{1}{r^2+1}\, dr\right)$

$$\int r\,dr - 2\int dr + 2\left(\int \frac{r}{r^2+1}\,dr - \int \frac{1}{r^2+1}\,dr\right)$$

Sean: $u = r^2+1 \quad v = r$

$du = 2r\,dr \quad dv = dr$

$$\int r\,dr - 2\int dr + 2\left(\frac{1}{2}\int \frac{du}{u} - \int \frac{dv}{v^2+1}\right)$$

$$\frac{r^2}{2} - 2r + 2\left(\frac{\ln|u|}{2} - \tan^{-1}(v)\right) + c$$

$$\frac{r^2}{2} - 2r + \ln|r^2+1| - 2\tan^{-1}(r) + c$$

g) $$\int \frac{dy}{(y-2)\sqrt{y^2-4y+3}}$$

$$\int \frac{dy}{(y-2)\sqrt{y^2-4y+4-1}}$$

$$\int \frac{dy}{(y-2)\sqrt{(y-2)^2-1}} \Longrightarrow \quad u = y-2 \quad a = 1, \quad du = dy$$

OK

$$\int \frac{du}{u\sqrt{u^2-a^2}}$$

$$\frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + c$$

$$\sec^{-1}(y-2) + c$$

h) $$\int_0^{\frac{3\sqrt{2}}{4}} \frac{ds}{\sqrt{9-4s^2}}$$

$u^2 = 4s^2 \therefore u = 2s \therefore du = 2\,ds$

$a^2 = 9 \therefore a = 3$

$$\int_a^b \frac{\frac{du}{2}}{\sqrt{a^2-u^2}}$$

$$\frac{1}{2}\cdot \operatorname{sen}^{-1}\left(\frac{u}{a}\right)\Big]_a^b$$

$$\frac{1}{2}\cdot \operatorname{sen}^{-1}\left(\frac{2s}{3}\right)\Big]_0^{\frac{3\sqrt{2}}{4}}$$

$$\frac{1}{2}\cdot \operatorname{sen}^{-1}\left(\frac{2\sqrt{2}}{4}\right) - \frac{1}{2}\cdot \operatorname{sen}^{-1}(0)$$

$$\frac{45}{2} - 0$$

$$\frac{45}{2} \rightarrow \frac{\pi}{8}$$

i) $\int_{-2}^{2} \frac{dt}{4+3t^2}$

Sea $a^2 = 4$ $\quad u^2 = 3t^2$

$a = 2$ $\quad u = \sqrt{3}\,t$ OK $\therefore\ du = \sqrt{3}\,dt$

$\frac{1}{\sqrt{3}} \int_a^b \frac{du}{a^2+u^2}$

$\frac{1}{\sqrt{3}} \cdot \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right)\Big]_a^b$

$\frac{1}{\sqrt{3}} \cdot \frac{1}{2} \tan^{-1}\left(\frac{\sqrt{3}\,t}{2}\right)\Big]_{-2}^{2}$

$\frac{\tan^{-1}(\sqrt{3})}{2\sqrt{3}} - \frac{\tan^{-1}(-\sqrt{3})}{2\sqrt{3}}$

$\frac{60}{2\sqrt{3}} - \frac{-60}{2\sqrt{3}}$

$\frac{30}{\sqrt{3}} + \frac{30}{\sqrt{3}}$

$\frac{60}{\sqrt{3}} \rightarrow \frac{\pi}{3\sqrt{3}}$

j) $\int_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}} \frac{dy}{y\sqrt{9y^2-1}}$ $\quad a^2 = 1$ $\quad u^2 = 9y^2$

$a = 1$ $\quad u = 3y$ OK $\quad du = 3\,dy$

$\int_a^b \frac{du}{u\sqrt{u^2-a^2}}$

$\ln\left|\frac{3y}{1+\sqrt{1-9y^2}}\right| \Big]_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}}$

$\ln\left|\frac{3(-\frac{\sqrt{2}}{3})}{1+\sqrt{1-9(-\frac{\sqrt{2}}{3})^2}}\right| - \ln\left|\frac{3(-\frac{2}{3})}{1+\sqrt{1-9(-\frac{2}{3})^2}}\right|$

$-2.1213 - (-0.8660)$

-0.2618

$(1-2x)^2$

$-1+4x-4x$

K) $\int_{1/2}^{1} \frac{6dx}{\sqrt{3+4x-4x^2}}$

$\int_{1/2}^{1} \frac{6dx}{\sqrt{4-1+4x-4x^2}}$

$\int_{1/2}^{1} \frac{6dx}{\sqrt{4-(1-4x+4x^2)}}$

$\int_{1/2}^{1} \frac{6dx}{\sqrt{4-(1-2x)^2}}$ $\Rightarrow$ $a^2 = 4$ $\quad u = 1-2x$

$a = 2$ $\quad du = -2\,dx$

OK

$\frac{6}{-2} \int_a^b \frac{du}{\sqrt{a^2-u^2}}$

$$-3 \cdot \operatorname{sen}^{-1}\left(\frac{v}{a}\right)\Big]_0^b$$

$$-3 \cdot \operatorname{sen}^{-1}\left(\frac{1-2x}{2}\right)\Big]_{1/2}^{1}$$

$$-3 \cdot \operatorname{sen}^{-1}\left(-\frac{1}{2}\right) - \left(-3 \cdot \operatorname{sen}^{-1}(0)\right)$$

$$\frac{\pi}{2}$$

l) $$\int_2^4 \frac{2dy}{y^2-6y+10}$$

$$\int_2^4 \frac{2dy}{y^2-6y+9+1}$$

$$\int_2^4 \frac{2dy}{(y-3)^2+1} \qquad a=1 \qquad u=y-3, \quad du=dy$$

$$2\int_c^d \frac{du}{u^2+a^2}$$

$$2 \cdot \frac{1}{a} \cdot \tan^{-1}\left(\frac{u}{a}\right)\Big]_c^d$$

$$2 \cdot \tan^{-1}(y-3)\Big]_2^4$$

$$2\tan^{-1}(1) - 2\tan^{-1}(-1)$$

$$2 \cdot \frac{\pi}{4} - \left(2 \cdot \frac{-\pi}{4}\right)$$

$$\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)$$

$$\pi$$

m) $$\int_{\sqrt{2}}^{2} \frac{\sec^2(\sec^{-1}(x))}{x\sqrt{x^2-1}}\,dx \qquad u=\sec^{-1}(x), \quad du=\frac{dx}{x\sqrt{x^2-1}}$$

$$\int_a^b \sec^2(u)\,du$$

$$\tan(u)\Big]_a^b$$

$$\tan\left[\sec^{-1}(x)\right]\Big]_{\sqrt{2}}^{2}$$

No existe función para los parámetros dados

No

Se considera!

Fecha de Entrega: 1° de abril del 2019

Tarea 8: Integrales en las que interviene la función logaritmo

1.- Evalúa las siguientes integrales

a) $\int \frac{x\,dx}{2-x^2}$

Si $u = 2 - x^2$
$du = -2x\,dx$
$\therefore x\,dx = \frac{du}{-2}$

$-\frac{1}{2}\int \frac{du}{u}$

$-\frac{1}{2} \cdot \ln|u| + c$

$-\frac{1}{2} \cdot \ln|2-x^2| + c$

b) $\int \frac{3y^2}{5y^3-1}$

$u = 5y^3 - 1$, $du = 15y^2dy \therefore \frac{du}{5} = 3y^2dy$

$\frac{1}{5}\int \frac{du}{u}$

$\frac{1}{5} \cdot \ln|u| + c$

$\frac{1}{5} \cdot \ln|5y^3-1| + c$

c) $\int \frac{dt}{t\ln(t)}$

$u = \ln(t)$, $du = \frac{1}{t}dt$

$\int \frac{du}{u}$

$\ln|u| + c$

$\ln(|\ln|t|) + c$

d) $\int \frac{\text{sen}(3z)}{\cos(3z)-1}\,dz$

$u = \cos(3z) - 1$ $\quad du = -3\,\text{sen}(3z)\,dz$
$\frac{du}{-3} = \text{sen}(3z)\,dz$

$-\frac{1}{3}\int \frac{du}{u}$

$-\frac{1}{3} \cdot \ln|u| + c$

$-\frac{1}{3} \cdot \ln|\cos(3z) - 1| + c$

e) $\int [\cot(5x) + \csc(5x)]\, dx$

$\int \left[\frac{\cos(5x)}{\text{sen}(5x)} + \frac{1}{\text{sen}(5x)}\right] dx$

$\int \frac{\cos(5x)}{\text{sen}(5x)}\, dx + \int \frac{dx}{\text{sen}(5x)}$

$u = \text{sen}(5x)$ $\quad v = 5x$

$du = 5\cos(5x)\, dx$ $\quad dv = 5\, dx$

$\frac{du}{5} = \cos(5x)\, dx$ $\quad \frac{dv}{5} = dx$

$\frac{1}{5}\int \frac{du}{u} + \frac{1}{5}\int \csc(v)\, dv$

$\frac{1}{5} \cdot \ln|u| + \frac{1}{5} \cdot \ln|\csc v - \cot v| + c$

$\frac{1}{5} \cdot \ln|\cos(5x)[\csc(5x) - \cot(5x)]| + c$

f) $\int \frac{2 - 3\,\text{sen}(2r)}{\cos(2r)}\, dr$

$\int \frac{2}{\cos(2r)} - \frac{3\,\text{sen}(2r)}{\cos(2r)}\, dr$

$\int 2\sec(2r)\, dr - \int \frac{3\,\text{sen}(2r)}{\cos(2r)}\, dr$ $\quad u = \cos(2r)$ $\quad du = -\text{sen}(2r) \cdot 2\, dr$

$\int \sec(v)\, dv - \frac{3}{-2}\int \frac{du}{u}$

$\ln|\sec(v) + \tan(v)| + \frac{3}{2} \cdot \ln|u| + c$

$\ln|\sec(2r) + \tan(2r)| + \frac{3}{2} \cdot \ln|\cos(2r)| + c$

g) $\int \frac{\cos(3y) + 3}{\text{sen}(3y)}\, dy$

$\int \cot(3y)\, dy + \int \csc(3y) \cdot 3\, dy$ $\quad u = 3y$ $\quad du = 3\, dy$

$\frac{1}{3}\int \cot(u)\, du + \int \csc(u)\, du$

$\frac{1}{3} \cdot \ln|\text{sen}(u)| + \ln|\csc(u) - \cot(u)| + c$

$\frac{1}{3} \cdot \ln|\text{sen}(3y)| + \ln|\csc(3y) - \cot(3y)| + c$

h) $\int [\tan(2t) - \sec(2t)]\, dt$ $\quad u = 2t \quad du = 2\, dt$

$\frac{1}{2}\int \tan(u)\, du - \frac{1}{2}\int \sec(u)\, du$

$\frac{1}{2} \cdot \ln|\sec(u)| - \frac{1}{2} \cdot \ln|\sec(u) + \tan(u)| + c$

$\frac{1}{2} \cdot \ln\left|\frac{\sec(2t)}{\sec(2t) + \tan(2t)}\right| + c$

i) $\int \frac{\tan(\ln(r))}{r}\, dr$ $\quad u = \ln(r) \quad du = \frac{dr}{r}$

$\int \tan(u)\, du$

$\ln|\sec(u)| + c$

$\ln|\sec[\ln(r)]| + c$

j) $\int \frac{\sec(\sqrt{z})}{\sqrt{z}}\, dz$ $\quad u = \sqrt{z} \quad du = \frac{dz}{2\sqrt{z}}$

$2\int \sec(u)\, du$

$2 \cdot \ln|\sec(u) + \tan(u)| + c$

$2 \cdot \ln|\sec(\sqrt{z}) + \tan(\sqrt{z})| + c$

k) $\int_0^2 \frac{3s}{s^2+4}\, ds$ $\quad u = s^2 + 4 \quad du = 2s\, ds$

$\frac{3}{2}\int_a^b \frac{du}{u}$

$\frac{3}{2} \cdot \ln|u| \Big]_a^b$

$\frac{3}{2} \cdot \ln|s^2+4| \Big]_0^2$

$\frac{3 \cdot \ln|2^2+4|}{2} - \frac{3 \cdot \ln|4|}{2}$

$3.119 - 2.079$

1.0397

l) $\int_0^{\pi/2} \frac{\cos(t)}{1 + 2\text{sen}(t)}\, dt$ $\quad u = 1 + 2\text{sen}(t) \quad du = 2\cos(t)\, dt$

$\frac{1}{2}\int_a^b \frac{du}{u}$

$\frac{1}{2} \cdot \ln|1 + 2\text{sen}(t)| \Big]_0^{\pi}$

$\frac{\ln|1+2|}{2} - \frac{\ln|1|}{2}$

$0.549 - 0$

0.549

m) $\int_0^{\pi/6} [\tan(2x) + \sec(2x)]\, dx \qquad u = 2x \quad du = 2dx$

$\frac{1}{2}\int_a^b \tan u \, du + \int_a^b \sec u \, du$

$\frac{1}{2} \cdot \ln|\sec u| + \ln|\sec u + \tan u| \Big]_a^b$

$\frac{1}{2} \cdot \ln|\sec(2x)| + \ln|\sec(2x) + \tan(2x)| \Big]_0^{\pi/6}$

$\left[\frac{\ln(2) + \ln(2+\sqrt{3})}{2}\right] - \left[\frac{\ln(1) + \ln(1)}{2}\right]$

$1.005 - 0$

1.005

n) $\int_{\pi/12}^{\pi/6} [\cot(3y) + \csc(3y)]\, dy \qquad u = 3y \qquad du = 3dy$

$\frac{1}{3}\left[\ln|\text{sen}(3y)| + \ln|\csc(3y) - \cot(3y)|\right] \Big]_{\pi/12}^{\pi/6}$

0.409

o) $\int_2^4 \frac{dr}{r \ln^2(r)} \qquad u = \ln(r) \qquad du = \frac{dr}{r}$

$\int_a^b u^{-2}\, du$

$-\frac{1}{u}\Big]_a^b$

$-\frac{1}{\ln(r)}\Big]_2^4$

$-\frac{1}{\ln 4} + \frac{1}{\ln 2}$

$-0.721 + 1.443$

0.721

$z \quad \ln z \qquad \ln z + 1$

Fecha de Entrega: 4 de abril del 2019

Tarea 9: Integrales de funciones exponenciales

1 - Evalúa las siguientes integrales

a) $\int a^{nx}\,dx$ $\qquad u = nx \qquad du = n\,dx$

$\frac{1}{n}\int a^u\,du$

$\frac{1}{n}\cdot\frac{a^u}{\ln a} + c$

$\frac{a^{nx}}{n\cdot\ln a} + c$

b) $\int a^t e^t\,dt$

$\int (ae)^t$

$\frac{ae^t}{\ln(ae)}$

c) $\int 5^{x^4+2x}(2x^3+1)\,dx$ $\qquad u = x^4+2x \qquad du = 4x^3+2\,dx$

$\frac{1}{2}\int 5^{x^4+2x}(4x^3+2)\,dx$

$\frac{1}{2}\int 5^u\,du$

$\frac{1}{2}\cdot\frac{5^u}{\ln 5}$

$\frac{5^u}{2\cdot\ln 5}$

d) $\int a^{z\ln(z)}(\ln(z)+1)\,dz$ $\qquad u = z\ln z \qquad du = \ln(z)+1\,dz$

$\int a^u\,du$

$\frac{a^u}{\ln a} + c$

$\frac{a^{z\ln(z)}}{\ln(a)} + c$

e) $\int e^y\, 2^{e^y}\, 3^{e^y}\,dy$

$\int e^y (2\cdot 3)^{e^y}\,dy$

$\int e^y (6)^{e^y}\,dy$ $\qquad u = e^y \qquad du = e^y\,dy$

$\int 6^u\,du$

$\frac{6^u}{\ln 6} + c$

$\frac{6^{e^y}}{\ln 6} + c$

$\ln z^2$

f) $\int \frac{4^{\ln(\frac{1}{x})}}{x} \, dx$ $\quad u = \ln(\frac{1}{x}) \quad du = \frac{1}{x} \, dx$

$\int 4^u \, du$

$\frac{4^u}{\ln 4} + c$

$\frac{4^{\ln(\frac{1}{x})}}{\ln 4} + c$

g) $\int_1^2 5^{-2v} \, dv$ $\quad u = -2v \quad du = -2 \, dv$

$-\frac{1}{2} \int_a^b 5^u \, du$

$-\frac{1}{2} \cdot \frac{5^u}{\ln 5} \Big]_a^b$

$-\frac{5^{-2v}}{2 \cdot \ln 5} \Big]_1^2$

$-\frac{5^{-2 \cdot 2}}{2 \cdot \ln 5} + \frac{5^{-2 \cdot 1}}{2 \cdot \ln 5}$

$-0.000497 + 0.0124$

0.0119

h) $\int_{-1}^{1} 2^{3x-1} \, dx$ $\quad u = 3x - 1 \quad du = 3 \, dx$

$\frac{1}{3} \int_a^b 2^u \, du$

$\frac{1}{3} \cdot \frac{2^u}{\ln 2} \Big]_a^b$

$\frac{2^{3x-1}}{3 \ln 2} \Big]_{-1}^{1}$

$-\frac{2^{-4}}{3 \ln 2} + \frac{2^2}{3 \ln 2}$

$-0.0301 + 1.924$

1.894

i) $\int_{-1}^{2} 2^s \, ds$

$\frac{2^s}{\ln 2} \Big]_{-1}^{2}$

$\frac{2^2}{\ln 2} - \frac{2^{-1}}{\ln 2}$

$5.77 - 0.721$

5.049

j) $$\int_{-2}^{0} (3^t - 5^t)\, dt$$

$$\int_{-2}^{0} 3^t\, dt - \int_{-2}^{0} 5^t\, dt$$

$$\frac{3^t}{\ln 3} - \frac{5^t}{\ln 5} \Big|_{-2}^{0}$$

$$\left(\frac{1}{\ln 3} - \frac{1}{\ln 5}\right) - \left(\frac{3^{-2}}{\ln 3} - \frac{5^{-2}}{\ln 5}\right)$$

$$\frac{1 - 3^{-2}}{\ln 3} + \frac{5^{-2} - 1}{\ln 5}$$

$$0.809 - 0.596$$

$$0.213$$

k) $$\int_{0}^{\pi/2} 7^{\cos(\theta)} \operatorname{sen}(\theta)\, d\theta \qquad u = \cos(\theta) \quad du = -\operatorname{sen}\theta\, d\theta$$

$$-\int_{a}^{b} 7^u\, du$$

$$-\frac{7^u}{\ln 7}\Big|_{a}^{b}$$

$$-\frac{7^{\cos\theta}}{\ln 7}\Big|_{0}^{\pi/2}$$

$$-\frac{1}{\ln 7} - \left(-\frac{7}{\ln 7}\right)$$

$$3.083$$

l) $$\int_{2}^{4} x^{2x} (1 + \ln(x))\, dx \qquad u = x^{2x} \quad du = x^{2x}(2\ln x + 2)\, dx$$

$$\int_{a}^{b} \frac{du}{2}$$

$$\frac{1}{2}\int_{a}^{b} du$$

$$\frac{1}{2} \cdot u \Big|_{a}^{b}$$

$$\frac{1}{2} \cdot x^{2x} \Big|_{2}^{4}$$

$$\frac{4^8}{2} - \frac{2^4}{2}$$

$$32{,}768 - 8$$

$$32{,}760$$

Fecha de Entrega: 4 de abril del 2019

Tarea 10: Integrales de funciones hiperbólicas

1.- Evalúa las siguientes integrales

a) $\int \operatorname{senh}^4(x)\cosh(x)\,dx$ $\qquad u = \operatorname{senh} x \quad du = \cosh x\,dx$

$\int u^4\,du$

$\frac{u^5}{5} + c$

$\frac{\operatorname{senh}^5(x)}{5} + c$

b) $\int \frac{\cosh[\ln(t)]}{t}\,dt$ $\qquad u = \ln t \quad du = \frac{dt}{t}$

$\int \cosh u\,du$

$\operatorname{senh} u + c$

$\operatorname{senh}[\ln(t)] + c$

c) $\int \frac{1}{\cosh^2(3x)}\,dx$ $\qquad u = 3x \quad du = 3\,dx$

$\frac{1}{3}\int \operatorname{sech}^2 u\,du$

$\frac{\tanh(3x)}{3} + c$

d) $\int \operatorname{senh}(z)\sqrt{\cosh(z)}\,dz$ $\qquad u = \cosh z \quad du = \operatorname{senh} z\,dz$

$\int \sqrt{u}\,du$

$\frac{u^{3/2}}{3/2} + c$

$\frac{2\cosh(z)\sqrt{\cosh(z)}}{5} + c$

e) $\int \tanh^2(3y)\operatorname{sech}^2(3y)\,dy$ $\qquad u = \tanh(3y) \quad du = 3\operatorname{sech}^2(3y)\,dy$

$\frac{1}{3}\int u^2\,du$

$\frac{1}{3}\cdot\frac{u^3}{3} + c$

$\frac{\tanh^3(3y)}{9} + c$

f) $\int \frac{\operatorname{sech}^2(x)}{1-2\tanh(x)}\,dx$ $\quad u = 1-2\tanh(x) \quad du = -2\operatorname{sech}^2(x)$

$-\frac{1}{2}\int \frac{du}{u}$

$-\frac{1}{2}\cdot \ln|u| + c$

$-\frac{\ln|1-2\tanh(x)|}{2} + c$

g) $\int_0^{\ln(3)} \operatorname{sech}^2(v)\,dv$

$\tanh(v)\Big|_0^{\ln 3}$

$0.8 - 0$

0.8

h) $\int_0^1 \cosh(x)\,dx$

$\operatorname{senh}(x)\Big|_0^1$

$1.175 - 0$

1.175

i) $\int_0^{\ln(2)} \tanh(s)\,ds$

$\ln[\cosh(s)]\Big|_0^{\ln 2}$

$0.223 - 0$

0.223

j) $\int_1^2 t\operatorname{sech}^2(t^2)\,dt$

$\frac{1}{2}\cdot \tanh(t^2)\Big|_1^2$

$0.4997 - 0.381$

0.119

k) $\int_2^3 \operatorname{sech}^2(y)\tanh^5(y)\,dy$ $\quad u = \tanh(y) \quad du = \operatorname{sech}^2(y)\,dy$

$\frac{\tanh^6(y)}{6}\Big|_2^3$

$0.162 - 0.134$

0.028

Fecha de Entrega: 6 de mayo del 2019

Tarea 11: Formas indeterminadas y Regla de L'Hôpital

1.- En los siguientes ejercicios, indicar la forma indeterminada y utilizar la regla de l'Hôpital para evaluar el límite dado

a) $\lim_{x \to 3} \dfrac{x^2+2x-15}{2x^2-5x-3}$

Al evaluarse el límite:

$$\frac{3^2+2(3)-15}{2(3^2)-5(3)-3} = \frac{0}{0} \Rightarrow$$ Esta es su forma indeterminada

Como nos encontramos en el caso en el que $\lim_{x \to x_0} f(x_0) = 0$ y $\lim_{x \to x_0} f(x_0) = 0$ y queremos calcular $\lim_{x \to x_0} \dfrac{f(x)}{g(x)}$ que resulta $\dfrac{0}{0}$

Las condiciones para aplicar la ley de l'Hôpital son las necesarias, por tanto, el límite puede encontrarse:

$$\lim_{x \to 3} \frac{2x+2}{4x-5}$$

$$\frac{\lim_{x \to 3} 2x+2}{\lim_{x \to 3} 4x-5} \Rightarrow \text{evaluando: } \frac{2(3)+2}{4(3)-5} = \frac{8}{7}$$

b) $\lim_{y \to -\frac{1}{2}} \dfrac{2y^2-5y-3}{2y^2+5y+2}$

Al evaluarse el límite:

$$\frac{\frac{1}{2}+\frac{5}{2}-3}{\frac{1}{2}-\frac{5}{2}+2} = \frac{0}{0} \Rightarrow$$ Tiene la forma $\frac{0}{0}$ indeterminada

Las condiciones necesarias se cumplen para realizar ley de l'Hôpital, por tanto:

$$\lim_{y \to -\frac{1}{2}} \frac{4y-5}{4y+5} = \frac{\lim_{y \to -\frac{1}{2}} 4y-5}{\lim_{y \to -\frac{1}{2}} 4y+5} \xrightarrow[\text{evaluando}]{} \frac{4(-\frac{1}{2})-5}{4(-\frac{1}{2})+5} = \frac{-7}{3}$$

c) $\lim_{\theta \to 0} \dfrac{\tan(A\theta)}{\tan(B\theta)}$ con A, B constantes

Al evaluarse el límite:

$$\frac{\tan(A0)}{\tan(B0)} = \frac{\tan(0)}{\tan(0)} = \frac{0}{0} \Rightarrow$$ Tiene la forma indeterminada $\frac{0}{0}$

Se cumplen las condiciones para realizar ley de l'Hôpital, por tanto:

$$\lim_{\theta \to 0} \frac{A\sec^2(A\theta)}{B\sec^2(B\theta)} \xrightarrow[\text{evaluando}]{} \frac{A(1)}{B(1)} = \frac{A}{B}$$

d) $\lim_{t \downarrow 0} \dfrac{\tan(t)}{\dfrac{1}{\ln[\cos(t)]}}$

Al evaluarse el límite:

$$\frac{\tan(0)}{\dfrac{1}{\ln[\cos(0)]}} = \frac{0}{\left[\dfrac{1}{0}\right]}$$

El tipo de indeterminación buscado no se encuentra, pero algebraicamente puede resolverse

$$\frac{[\tan(t)]\{\ln[\cos(t)]\}}{1}$$

evaluando: $\tan(0) \cdot \ln[\cos(0)] = 0$

e) $\lim_{x \uparrow \infty} \dfrac{-x^2}{\dfrac{1}{\sqrt{x}}}$

El tipo de indeterminación no se encuentra, pero puede resolverse:

$$-x^2 \sqrt{x}$$

evaluando: $-\infty \cdot \infty = -\infty$

f) $\lim_{x \downarrow -\infty} x^3 e^{-x}$

evaluando: $\lim_{x \downarrow -\infty} x^3 e^{-x} = (-\infty)(\infty) = -\infty$

g) $\lim_{x \to 1} \left[\dfrac{x}{\ln(x)} - \dfrac{1}{\ln(x)}\right]$

$\lim_{x \to 1} \left[\dfrac{x-1}{\ln(x)}\right]$ evaluando: $\dfrac{0}{0}$ → Forma indeterminada

Se procede a aplicar la Regla de l'Hôpital:

$$\lim_{x \to 1} \frac{1}{\left[\dfrac{1}{x}\right]} = \lim_{x \to 1} x = 1$$

h) $\lim_{x \uparrow \infty} \dfrac{e^x}{x^2}$

evaluando: $\dfrac{\infty}{\infty}$ → Forma indeterminada

Se procede a aplicar la Regla de l'Hôpital:

$\lim_{x \uparrow \infty} \dfrac{e^x}{2x} = \dfrac{\infty}{\infty}$ → Nueva forma indeterminada

Se vuelve a aplicar la Regla:

$$\lim_{x \uparrow \infty} \frac{e^x}{2} = \frac{\infty}{2} = \infty$$

i) $\lim_{x \to 1} (x)^{\left(\frac{1}{1-x^2}\right)}$

evaluando: $(1)^{\left(\frac{1}{0}\right)} = 1^{\infty}$

Para resolver la indeterminación anterior se hace lo siguiente:

$$\lim_{x \to 1} (x)^{\left(\frac{1}{1-x^2}\right)} = \lim_{x \to 1} e^{\ln\left[(x)^{\left(\frac{1}{1-x^2}\right)}\right]} = \lim_{x \to 1} e^{\left(\frac{1}{1-x^2}\right) \cdot \ln(x)}$$

Como la función exponencial es continua, el límite puede entrar al exponente:

$$e^{\lim_{x \to 1} \left(\frac{1}{1-x^2}\right) \cdot \ln(x)}$$

Enfocándose únicamente en el exponente:

$$\lim_{x \to 1} \left(\frac{1}{1-x^2}\right) \cdot \ln(x) = \lim_{x \to 1} \frac{\ln(x)}{1-x^2} = \frac{0}{0} \rightarrow \text{forma indeterminada}$$

Se aplica la Regla de l'Hôpital:

$$\lim_{x \to 1} \frac{\frac{1}{x}}{-2x} = \frac{1}{-2x^2} = -\frac{1}{2}$$

Se acaba de resolver el exponente, por tanto se sustituye lo obtenido en el exponente:

$$e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

j) $\lim_{x \to 1} (x-1)^{1+x^2}$

evaluando: $0^2 = 0$

k) $\lim_{x \to 1} \left[\frac{1}{\ln(x)} - \frac{1}{x}\right]$

evaluando: $\frac{1}{0} - 1 = \infty - 1 = \infty$

Fecha de Entrega: 6 de mayo del 2019

Tarea 12: Integrales Impropias (Primera Parte)

1.- Evaluar la integral impropia dada, o probar que diverge

a) $\int_{-\infty}^{3} e^{2x}\,dx$

$\lim_{t \to -\infty} \int_{t}^{3} e^{2x}\,dx$ $\quad u = 2x \quad du = 2\,dx$

$\lim_{t \to -\infty} \frac{1}{2}\int_{a}^{b} e^{u}\,du$

$\lim_{t \to -\infty} \left.\frac{e^{2x}}{2}\right|_{t}^{3}$

$\lim_{t \to -\infty} \left(\frac{e^{6}}{2} - \frac{e^{2t}}{2}\right)$

$\frac{e^{6}}{2} - 0$

$\frac{e^{6}}{2}$

b) $\int_{-\infty}^{0} \frac{x}{(x^2+9)^2}\,dx$

$\lim_{t \to -\infty} \int_{t}^{0} \frac{x}{(x^2+9)^2}\,dx$ $\quad u = x^2+9 \quad du = 2x\,dx$

$\lim_{t \to -\infty} \frac{1}{2}\int_{a}^{b} u^{-2}\,du$

$\lim_{t \to -\infty} \left.\frac{1}{2}\cdot\frac{u^{-1}}{-1}\right|_{a}^{b}$

$\lim_{t \to -\infty} \left.-\frac{1}{2(x^2+9)}\right|_{t}^{0}$

$\lim_{t \to -\infty} \left(-\frac{1}{2(0^2+9)} + \frac{1}{2(t^2+9)}\right)$

$-\frac{1}{18} + \frac{1}{2t^2+18}$

$-\frac{1}{18} + 0$

$-\frac{1}{18}$

c) $\int_{-\infty}^{1} \frac{1}{\sqrt[4]{x}}\, dx$

$\lim_{t \to -\infty} \int_{t}^{1} \frac{1}{\sqrt[4]{x}}\, dx$

$\lim_{t \to -\infty} \left. \frac{4\sqrt[4]{x^3}}{3} \right|_{t}^{1}$

$\lim_{t \to -\infty} \left(\frac{4(1)}{3} - \frac{4\sqrt[4]{t^3}}{3} \right) \rightarrow$ No es posible calcular la raiz cuarta de números negativos

La integral diverge

d) $\int_{-\infty}^{0} \frac{1}{x^2 - 3x + 2}\, dx$

$\int_{-\infty}^{0} \frac{1}{(x-2)(x-1)}\, dx = \int_{-\infty}^{0} \frac{1 \cdot 1}{(x-2)(x-1)}\, dx$

$\lim_{t \to -\infty} \left[\int_{t}^{0} \frac{dx}{x-2} - \int_{t}^{0} \frac{dx}{x-1} \right]$

$\lim_{t \to -\infty} \left[\ln|x-2| \Big|_{t}^{0} - \ln|x-1| \Big|_{t}^{0} \right]$

$\lim_{t \to -\infty} \left[\ln 2 - \ln|t-2| - \ln 1 + \ln|t-1| \right]$

La integral diverge

e) $\int_{1}^{\infty} \frac{\ln(x)}{x}\, dx$ $\qquad u = \ln(x) \qquad du = \frac{dx}{x}$

$\lim_{t \to \infty} \ln^2(x) \Big|_{1}^{\infty}$

$\ln^2(\infty) - 0$

∞

La integral diverge

f) $\int_{1}^{\infty} \frac{\ln(x)}{x^2}\, dx$

$\lim_{t \to \infty} \int_{1}^{t} \frac{\ln(x)}{x^2}\, dx$ $\qquad u = \ln(x) \qquad du = \frac{dx}{x}$

$dv = \frac{dx}{x^2} \qquad v = -\frac{1}{x}$

$-\frac{\ln(x)}{x} - \int -\frac{dx}{x^2}$

$-\frac{\ln(x)}{x} - \frac{1}{x}$

$\lim_{t \to \infty} \left[\left(-\frac{\ln(t)}{t} - \frac{1}{t} \right) - \left(-\frac{\ln 1}{1} - 1 \right) \right]$

$\lim_{t\to\infty} \left[\left(-\frac{\ln(t)}{t} - \frac{1}{t}\right) + 1\right]$

$\lim_{t\to\infty} -\frac{\ln(t)}{t} + 1$

Como existe la indeterminación $\frac{\infty}{\infty}$, se aplica l'Hôpital

$-\frac{\frac{1}{t}}{1} + 1$

$-\infty + 1$

$-\infty$

La integral diverge

g) $\int_1^\infty \left[\frac{1}{x} - \frac{1}{x+1}\right] dx$

$\lim_{t\to\infty} \int_1^t \left[\frac{1}{x} - \frac{1}{x+1}\right] dx$

$\lim_{t\to\infty} \left[\ln|x| - \ln|x+1|\right]\Big|_1^t$

$\lim_{t\to\infty} \left[\left(\ln|t| - \ln|t+1|\right) - \left(\ln 1 - \ln 2\right)\right]$

$\infty - \infty - 0 + \ln 2$

$-\infty$

La integral diverge

h) $\int_2^\infty t e^{-t}\, dt$

$\lim_{t\to\infty} \int_2^t t e^{-t}\, dt$

$u = t \qquad du = dt$

$dv = e^{-t}\, dt \qquad v = -e^{-t}$

$t \cdot -e^{-t} - \int -e^{-t}\, dt$

$-e^{-t} \cdot t - e^{-t} \Big|_2^\infty$

$\lim_{t\to\infty} \{[-e^{-t}(t+1)] - [-e^{-2}(3)]\}$

$0 - (-e^{-2}(3))$

0.406

i) $\int_{-\infty}^\infty e^{-x}\, dx$

$\int_0^\infty e^{-x}\, dx + \int_{-\infty}^0 e^{-x}\, dx$

$\lim_{s\to\infty} \int_0^s e^{-x}\, dx + \lim_{t\to-\infty} \int_t^0 e^{-x}\, dx$

$\lim_{s\to\infty} -e^{-x} \Big|_0^s + \lim_{t\to-\infty} -e^{-x} \Big|_t^0$

$\lim_{s\to\infty}\left[(-e^{-s})-(-e^{-0})\right]+\lim_{t\to-\infty}\left[(-e^{-0})-(-e^{-t})\right]$

$[0+1]+[1+\infty]$

∞

La integral diverge

j) $\int_{-\infty}^{\infty}\frac{x}{(x^2+1)^{3/2}}\,dx \qquad u=x^2+1 \qquad du=2x\,dx$

$\frac{1}{2}\int_{-\infty}^{\infty}u^{-3/2}\,du$

$\frac{1}{2}\cdot\frac{u^{-1/2}}{-\frac{1}{2}}\Big|_{-\infty}^{\infty}$

$\frac{1}{\sqrt{x^2+1}}\Big|_{-\infty}^{\infty}$

$\lim_{s\to\infty}\frac{1}{\sqrt{x^2+1}}\Big|_0^s+\lim_{t\to-\infty}\frac{1}{\sqrt{x^2+1}}\Big|_t^0$

$\lim_{s\to\infty}\left[\left(\frac{1}{\sqrt{s^2+1}}\right)-\left(\frac{1}{\sqrt{1}}\right)\right]+\lim_{t\to-\infty}\left[\left(\frac{1}{\sqrt{1}}\right)-\left(\frac{1}{\sqrt{t^2+1}}\right)\right]$

$[0-1]+[1-0]$

0

k) $\int_{-\infty}^{\infty}xe^{-x^2}\,dx \qquad u=-x^2 \qquad du=-2x\,dx$

$-\frac{1}{2}\int_{-\infty}^{\infty}e^{u}\,du$

$-\frac{e^{-x^2}}{2}\Big|_{-\infty}^{\infty}$

$\lim_{s\to\infty}-\frac{e^{-x^2}}{2}\Big|_0^s+\lim_{t\to-\infty}-\frac{e^{-x^2}}{2}\Big|_t^0$

$[(0)-(-\frac{1}{2})]+[(-\frac{1}{2})-(0)]$

0

l) $\int_{-\infty}^{\infty}\frac{x}{x^2+1}\,dx \qquad u=x^2+1 \qquad du=2x\,dx$

$\frac{1}{2}\int_{-\infty}^{\infty}\frac{du}{u}$

$\frac{\ln|x^2+1|}{2}\Big|_{-\infty}^{\infty}=\lim_{s\to\infty}\frac{\ln|x^2+1|}{2}\Big|_0^s+\lim_{t\to-\infty}\frac{\ln|x^2+1|}{2}\Big|_t^0$

$\cancel{\frac{\ln|(\infty)^2+1|}{2}}-\cancel{\frac{\ln 1}{2}}+\cancel{\frac{\ln 1}{2}}-\cancel{\frac{\ln|(-\infty)^2+1|}{2}}$

0

Fecha de Entrega: 6 de mayo del 2019

Tarea 13: Integrales Impropias (Segunda Parte)

1- Evaluar la integral impropia dada, o probar que diverge

a) $\int_0^5 \frac{1}{x}\,dx$

$\ln|x| \Big|_0^5 = \lim_{s \to 0} \ln|x| \Big|_s^5$

$\ln 5 - \infty$

La integral diverge

b) $\int_0^2 \frac{1}{\sqrt{2-x}}\,dx$ $\qquad u = 2-x \qquad du = -dx$

$\int_0^2 u^{-1/2} - du$

$-\frac{u^{1/2}}{\frac{1}{2}} \Big|_a^b = -2\sqrt{2-x}\,\Big|_0^2$

$\lim_{t \to 2} -2\sqrt{2-x}\,\Big|_0^t$

$\lim_{t \to 2} \left[(-2\sqrt{2-t}) - (-2\sqrt{2})\right]$

$0 + 2\sqrt{2} = 2\sqrt{2}$

c) $\int_1^3 \frac{1}{(x-1)^2}\,dx$

$-\frac{1}{(x-1)} \Big|_1^3$

$\lim_{t \to 1} -\frac{1}{x-1} \Big|_t^3$

$\lim_{t \to 1} -\frac{3}{t-1} + \frac{3}{2} = -\infty$

La integral diverge

d) $\int_{-1}^1 \frac{1}{\sqrt[3]{x^5}}\,dx$

$-\frac{3}{2x^{2/3}} \Big|_{-1}^1 = -\frac{3}{2x^{2/3}} \Big|_0^1 + \left(-\frac{3}{2\sqrt[3]{x^2}} \Big|_{-1}^0\right)$

$\lim_{t \to 0} \left(-\frac{3}{2\sqrt[3]{x^2}} \Big|_t^1 - \frac{3}{2\sqrt[3]{x^2}} \Big|_{-1}^t\right)$

$\lim_{t \to 0} \left(-\frac{3}{2} + \frac{3}{2\sqrt[3]{t^2}} - \frac{3}{2\sqrt[3]{t^2}} + \frac{3}{2}\right)$

0

e) $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx$

$\frac{3(x-1)^{2/3}}{2}\Big|_0^2$

$\lim_{t\to 1}\left(\frac{3(x-1)^{2/3}}{2}\Big|_t^2 + \frac{3(x-1)^{2/3}}{2}\Big|_0^t\right)$

$\lim_{t\to 1}\left(\frac{3}{2} - \frac{3(t-1)^{2/3}}{2} + \frac{3(t-1)^{2/3}}{2} - \frac{3}{2}\right)$

0

f) $\int_0^3 \frac{1}{\sqrt[3]{(x-1)^2}}\,dx$

$3(x-1)^{1/3}\Big|_0^2$

$\lim_{t\to 1}\left(3\sqrt[3]{x-1}\Big|_t^2 + 3\sqrt[3]{x-1}\Big|_0^t\right)$

$\lim_{t\to 1}\left(3 - \cancel{3\sqrt[3]{t-1}} + \cancel{3\sqrt[3]{t-1}} - (3(-1))\right)$

$3+3=6$

g) $\int_0^{\pi} \frac{\cos(x)}{\sqrt{1-\text{sen}(x)}}\,dx$ $\quad u = 1-\text{sen}\,x \quad du = \cos x\,dx$

$-2\sqrt{1-\text{sen}(x)}\Big|_0^{\pi}$

$\lim_{t\to\pi} -2\sqrt{1-\text{sen}(x)}\Big|_0^t$

$\lim_{t\to\pi}\left(-2\sqrt{1-\text{sen}(t)} + 2\sqrt{\text{sen}(0)}\right)$

$(-2(0)+2(0))$

0

h) $\int_0^{\pi} \frac{\text{sen}(x)}{1+\cos(x)}\,dx$ $\quad u = 1+\cos x \quad du = -\text{sen}\,x$

$-\ln|1+\cos(x)|\Big|_0^{\pi}$

$\lim_{t\to\pi}\left(-\ln|0| + \ln 1\right)$

La integral diverge

i) $\int_{-2}^0 \frac{1}{1+x^2}\,dx$

$\tan^{-1}(x)\Big|_{-2}^0$

$\tan^{-1} 0 - \tan^{-1} -2$

$0-(-1.107)$

1.107

j) $\int_0^3 \frac{1}{t^2-1}\,dt$

$\frac{1}{2}\cdot\ln\left(\frac{t-1}{t+1}\right)\Big|_0^3 = \lim_{t\to 1}\left[\left(\frac{1}{2}\cdot\ln\frac{1}{2} - \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right)\right) + \frac{1}{2}\ln\left(\frac{t-1}{t+1}\right) - \frac{1}{2}\ln -1\right]$

La integral diverge

k) $\int_0^1 \left(\frac{1}{x} + \frac{1}{1-x} \right) dx$

$\ln|x| - \ln|1-x| \Big|_0^1$

$\lim_{t \to 0} (\ln|x| - \ln|1-x| \Big|_t^1)$

$\lim_{t \to 0} (\ln|1| - \ln 0 - \ln t + \ln|1-t|)$

La integral diverge

l) $\int_0^1 \left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{1-x}} \right) dx$

$2\sqrt{x} - 2\sqrt{1-x} \Big|_0^1$

$\lim_{t \to 0} (2\sqrt{x} - 2\sqrt{1-x} \Big|_t^1)$

$\lim_{t \to 0} (2 - 0 - 2\sqrt{t} + 2\sqrt{1-t})$

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Fecha de Entrega: 6 de mayo del 2019

Inversas
Logarítmicas
Algebraicas udv $uv - \int v\,du$
Trigonométricas
Exponenciales

Tarea 14: Integración por Partes

1.- Evaluar las siguientes integrales:

a) $\int y^2 \tan^{-1}(y)\, dy$

Para integrar por partes, se propone

$u = \tan^{-1}(y) \quad \therefore \quad du = \dfrac{dy}{1+y^2}$

$dv = y^2\, dy \quad \therefore \quad v = \dfrac{y^3}{3}$

$\int u\,dv = uv - \int v\,du$

$$\frac{y^3 \tan^{-1}(y)}{3} - \int \frac{y^3}{3(1+y^2)}\,dy$$

$$\frac{y^3 \tan^{-1}(y)}{3} - \frac{1}{3}\int \frac{y^3}{y^2+1}\,dy$$

$$\frac{y^3 \tan^{-1}(y)}{3} - \frac{1}{3}\int \frac{y^3+y-y}{y^2+1}\,dy$$

$$\frac{y^3 \tan^{-1}(y)}{3} - \frac{1}{3}\int \frac{y^3+y}{y^2+1} - \frac{y}{y^2+1}\,dy$$

$$\frac{y^3 \tan^{-1}(y)}{3} - \frac{1}{3}\left[\int \frac{y\cancel{(y^2+1)}}{\cancel{y^2+1}}\,dy - \int \frac{y}{y^2+1}\,dy\right] + c$$

$$\frac{y^3 \tan^{-1}(y)}{3} - \frac{1}{3}\left[\frac{y^2}{2} - \int \frac{y}{y^2+1}\,dy\right] \longrightarrow u = y^2+1 \quad du = 2y\,dy$$

$$\frac{y^3 \tan^{-1}(y)}{3} - \frac{1}{3}\left[\frac{y^2}{2} - \frac{\ln(y^2+1)}{2}\right] + c$$

$$\frac{y^3 \tan^{-1}(y)}{3} - \frac{y^2}{6} + \frac{\ln(y^2+1)}{6} + c$$

b) $\int [z \ln(z)]^2\, dz$

$\int z^2 \ln^2(z)\, dz$

Para integrar por partes, se propone

$u = \ln^2(z) \qquad du = \dfrac{2\ln(z)}{z}$

$dv = z^2\, dz \qquad v = \dfrac{z^3}{3}$

$\int u\,dv = uv - \int v\,du$

$$\ln^2(z) \cdot \frac{z^3}{3} - \int \frac{z^3 \cdot 2\ln(z)}{3z}\,dz$$

$$\frac{z^3 \ln^2(z)}{3} - \frac{2}{3}\int z^2 \ln(z)\,dz$$

Se propone volver a integrar por partes con las siguientes:

$u = \ln(z) \qquad du = \dfrac{1}{z}\,dz$

$dv = z^2\, dz \qquad v = \dfrac{z^3}{3}$

$$\frac{z^3 \ln^2(z)}{3} - \frac{2}{3}\left[\frac{z^3 \ln(z)}{3} - \int \frac{z^3}{3z}\,dz\right]$$

$$\frac{z^3\ln^2(z)}{3}-\frac{2}{3}\left[\frac{z^3\ln(z)}{3}-\frac{1}{3}\int z^2\,dz\right]$$

$$\frac{z^3\ln^2(z)}{3}-\frac{2}{3}\left[\frac{z^3\ln(z)}{3}-\frac{1}{3}\left(\frac{z^3}{3}\right)\right]+c$$

$$\frac{z^3\ln^2(z)}{3}-\frac{2z^3\ln(z)}{9}+\frac{2z^3}{27}+c$$

c) $\int x^3 e^{-4x}\,dx$

Para integrar por partes, se propone

$u=x^3$ $\quad du=3x^2\,dx$

$dv=e^{-4x}\,dx$ $\quad v=\frac{e^{-4x}}{-4}$

$\int u\,dv=uv-\int v\,du$

$$-\frac{x^3e^{-4x}}{4}-\int-\frac{3x^2e^{-4x}}{4}\,dx$$

$$-\frac{x^3e^{-4x}}{4}+\frac{3}{4}\int x^2e^{-4x}\,dx$$

Para integrar por partes, se propone

$u=x^2$ $\quad du=2x\,dx$

$dv=e^{-4x}\,dx$ $\quad v=\frac{e^{-4x}}{-4}$

$\int u\,dv=uv-\int v\,du$

$$-\frac{x^3e^{-4x}}{4}+\frac{3}{4}\left[-\frac{x^2e^{-4x}}{4}-\int-\frac{2xe^{-4x}}{4}\,dx\right]$$

$$-\frac{x^3e^{-4x}}{4}+\frac{3}{4}\left[-\frac{x^2e^{-4x}}{4}+\frac{1}{2}\int xe^{-4x}\,dx\right]$$

Para integrar por partes, se propone

$u=x$ $\quad du=dx$

$dv=e^{-4x}\,dx$ $\quad v=\frac{e^{-4x}}{-4}$

$\int u\,dv=uv-\int v\,du$

$$-\frac{x^3e^{-4x}}{4}+\frac{3}{4}\left[-\frac{x^2e^{-4x}}{4}+\frac{1}{2}\left(-\frac{xe^{-4x}}{4}-\int\frac{e^{-4x}}{4}\,dx\right)\right]$$

$$-\frac{x^3e^{-4x}}{4}+\frac{3}{4}\left[-\frac{x^2e^{-4x}}{4}+\frac{1}{2}\left(-\frac{xe^{-4x}}{4}+\frac{e^{-4x}}{16}\right)\right]+c$$

$$-\frac{xe^{-4x}}{4}+\frac{3}{4}\left[-\frac{x^2e^{-4x}}{4}-\frac{xe^{-4x}}{8}+\frac{e^{-4x}}{32}\right]+c$$

$$-\frac{xe^{-4x}}{4}-\frac{3x^2e^{-4x}}{16}-\frac{3xe^{-4x}}{32}+\frac{3e^{-4x}}{128}+c$$

d) $\int x^2 \cos\left(\frac{x}{2}\right) dx$

Para integrar por partes, se propone

$u = x^2 \quad du = 2x\,dx$

$dv = \cos\left(\frac{x}{2}\right) dx \quad v = 2\,\text{sen}\left(\frac{x}{2}\right)$

$\int u\,dv = uv - \int v\,du$

$2x^2\,\text{sen}\left(\frac{x}{2}\right) - \int 4x\,\text{sen}\left(\frac{x}{2}\right) dx$

$2x^2\,\text{sen}\left(\frac{x}{2}\right) - 4\int x\,\text{sen}\left(\frac{x}{2}\right) dx$

Para integrar por partes, se propone

$u = x \quad du = dx$

$dv = \text{sen}\left(\frac{x}{2}\right) dx \quad v = -2\cos\left(\frac{x}{2}\right)$

$\int u\,dv = uv - \int v\,du$

$2x^2\,\text{sen}\left(\frac{x}{2}\right) - 4\left[-2x\cos\left(\frac{x}{2}\right) - \int -2\cos\left(\frac{x}{2}\right) dx\right]$

$2x^2\,\text{sen}\left(\frac{x}{2}\right) - 4\left[-2x\cos\left(\frac{x}{2}\right) + 2\int \cos\left(\frac{x}{2}\right) dx\right]$

$2x^2\,\text{sen}\left(\frac{x}{2}\right) - 4\left[-2x\cos\left(\frac{x}{2}\right) + 4\,\text{sen}\left(\frac{x}{2}\right)\right] + c$

$2x^2\,\text{sen}\left(\frac{x}{2}\right) + 8\cos\left(\frac{x}{2}\right) - 16\,\text{sen}\left(\frac{x}{2}\right) + c$

e) $\int e^{ax} \cos(\beta x)\, dx$

Por fórmula:

$$\frac{e^{ax}\left[a\cos(\beta x) + \beta\,\text{sen}(\beta x)\right]}{a^2 + \beta^2} + c$$

f) $\int x^3 e^{x^2} dx$

$\int x^2 (x e^{x^2})\, dx$

Para integrar por partes, se propone

$u = x^2 \quad du = 2x\,dx$

$dv = x e^{x^2} dx \quad v = \frac{e^{x^2}}{2}$

$\int u\,dv = uv - \int v\,du$

$\frac{x^2 e^{x^2}}{2} - \int \frac{2x e^{x^2}}{2} dx$

$\frac{x^2 e^{x^2}}{2} - \int x e^{x^2} dx$

$\frac{x^2 e^{x^2}}{2} - \frac{e^{x^2}}{2} + c$

$\frac{x^2 e^{x^2} - e^{x^2}}{2} + c$

g) $\int_0^1 (z^2+1)\,e^{-z}\,dz$

Para integrar por partes, se propone

$u = z^2+1 \qquad du = 2z\,dz$

$dv = e^{-z}dz \qquad v = -e^{-z}$

$\int u\,dv = uv - \int v\,du$

$-(z^2+1)e^{-z} - \int -2z\,e^{-z}\,dz$

$-(z^2+1)e^{-z} + 2\int z\,e^{-z}\,dz$

Para integrar por partes se propone

$u = z \qquad du = dz$

$dv = e^{-z}dz \qquad v = -e^{-z}$

$\int u\,dv = uv - \int v\,du$

$-(z^2+1)e^{-z} + 2\left(-ze^{-z} - \int -e^{-z}dz\right)$

$-(z^2+1)e^{-z} + 2\left(-ze^{-z} - e^{-z}\right)$

$-(z^2+1)e^{-z} - 2ze^{-z} - 2e^{-z}\Big|_0^1$

$-2.207 - (-3)$

0.793

h) $\int_4^9 \frac{\ln(y)}{\sqrt{y}}\,dy$

Para integrar por partes se propone

$u = \ln(y) \qquad du = \frac{1}{y}dy$

$dv = y^{-1/2}dy \qquad v = 2\sqrt{y}$

$\int u\,dv = uv - \int v\,du$

$2\sqrt{y}\,\ln(y) - \int \frac{2\sqrt{y}}{y}\,dy$

$2\sqrt{y}\,\ln(y) - 2\int \frac{dy}{\sqrt{y}}$

$2\sqrt{y}\,\ln(y) - 4\sqrt{y}\Big|_4^9$

$1.183 - (-2.455)$

3.638

i) $\int_0^{2\pi} t^2\,\text{sen}(2t)\,dt$

Para integrar por partes se propone

$u = t^2 \qquad du = 2t\,dt$

$dv = \text{sen}(2t)\,dt \qquad v = \frac{-\cos(2t)}{2}$

$\int u\,dv = uv - \int v\,du$

$-\frac{t^2\cos(2t)}{2} - \int \frac{-2t\cos(2t)}{2}\,dt$

$-\frac{t^2\cos(2t)}{2} + \int t\cos(2t)\,dt$

Para integrar por partes, se propone

$u = t \qquad du = dt$

$dv = \cos(2t)\,dt \qquad v = \frac{\text{sen}(2t)}{2}$

$\int u\,dv = uv - \int v\,du$

$-\frac{t^2\cos(2t)}{2} + \left[\frac{t\,\text{sen}(2t)}{2} - \int \frac{\text{sen}(2t)}{2}\,dt\right]$

$-\frac{t^2\cos(2t)}{2} + \left[\frac{t\,\text{sen}(2t)}{2} + \frac{\cos(2t)}{4}\right]$

$-\frac{t^2\cos(2t)}{2} + \frac{t\,\text{sen}(2t)}{2} + \frac{\cos(2t)}{4}\Big|_0^{2\pi}$

$-19.489 - 0.25$

-19.739

j) $\int_1^{\sqrt{3}} \tan^{-1}\left(\frac{1}{r}\right) dr$

Para integrar por partes, se propone

$u = \tan^{-1}\left(\frac{1}{r}\right) \qquad du = \frac{1}{1+\frac{1}{r^2}} \cdot -\frac{1}{r^2}\,dr$

$dv = dr \qquad v = r$

$\int u\,dv = uv - \int v\,du$

$r\tan^{-1}\left(\frac{1}{r}\right) - \int -r \cdot \frac{1}{r^2+1}\,dr$

$r\tan^{-1}\left(\frac{1}{r}\right) + \int \frac{r}{r^2+1}\,dr$

$r\tan^{-1}\left(\frac{1}{r}\right) + \frac{\ln(r^2+1)}{2}\Big|_1^{\sqrt{3}}$

$1.600 - 1.132$

0.468

k) $\int_{-\pi}^{\pi} e^{\theta}\cos\theta\,d\theta$

Para integrar por partes, se propone

$u = \cos\theta \qquad du = -\text{sen}\,\theta\,d\theta$

$dv = e^{\theta}\,d\theta \qquad v = e^{\theta}$

$\int u\,dv = uv - \int v\,du$

$e^{\theta}\cos\theta - \int e^{\theta} \cdot -\text{sen}\,\theta\,d\theta$

$e^{\theta}\cos\theta + \int e^{\theta}\,\text{sen}\,\theta\,d\theta$

Para integrar por partes, se propone

$u = \text{sen}\,\theta \qquad du = \cos\theta\,d\theta$

$dv = e^{\theta}\,d\theta \qquad v = e^{\theta}$

$\int u\,dv = uv - \int v\,du$

$e^{\theta}\cos\theta + (\text{sen}\,\theta \cdot e^{\theta} - \int e^{\theta}\cos\theta\,d\theta)$

$e^{\theta} \cos\theta + e^{\theta} \operatorname{sen}\theta - \int e^{\theta} \cos\theta \, d\theta$

Por propiedad de transitividad, podemos establecer la igualdad:

$\int e^{\theta} \cos\theta \, d\theta = e^{\theta} \cos\theta + e^{\theta} \operatorname{sen}\theta - \int e^{\theta} \cos\theta \, d\theta$

$2\int e^{\theta} \cos\theta \, d\theta = e^{\theta} \cos\theta + e^{\theta} \operatorname{sen}\theta$

$= \dfrac{e^{\theta} \cos\theta + e^{\theta} \operatorname{sen}\theta}{2}$

$-0.022 - (-11.570)$

11.549

En los siguientes ejercicios, realizar primero sustitución

l) $\int e^{\sqrt{3s+1}} \, ds$

Primero se sustituye:

$p = \sqrt{3s+1}$

$dp = \dfrac{3}{2\sqrt{3s+1}} \, ds \Longrightarrow \dfrac{2\sqrt{3s+1}}{3} \, dp = ds \Longrightarrow \dfrac{2p\,dp}{3} = ds$

$\int e^{p} \cdot \dfrac{2p\,dp}{3} = \dfrac{2}{3}\int p e^{p} \, dp$

Ahora, para integrar por partes, se propone:

$u = p \qquad du = dp$

$dv = e^{p} dp \qquad v = e^{p}$

$\int u\,dv = uv - \int v\,du$

$pe^{p} - \int e^{p} dp$

$\dfrac{2}{3}\left[pe^{p} - \int e^{p} dp\right]$

$\dfrac{2}{3}\left[pe^{p} - e^{p}\right] + c$

$\dfrac{2}{3}\left[\sqrt{3s+1}\, e^{\sqrt{3s+1}} - e^{\sqrt{3s+1}}\right] + c$

$\dfrac{2\sqrt{3s+1}\, e^{\sqrt{3s+1}} - 2e^{\sqrt{3s+1}}}{3} + c$

m) $\int \ln(y + y^2) \, dy$

$\int \ln[y(1+y)] \, dy$

$\int [\ln y + \ln(1+y)] \, dy$

$\int \ln y \, dy + \int \ln(1+y) \, dy$

$[y \ln y - y] + [(1+y)\ln|1+y| - (1+y)] + c$

n) $\int \operatorname{sen}[\ln(z)] \, dz$

Primero se sustituye

$p = \ln(z)$

$dp = \dfrac{dz}{z} \Longrightarrow z\,dp = dz$

Esto puede reescribirse como:
$\int z \operatorname{sen}(p)\, dp$
Pero, para efectos de completar la sustitución:
Como $p = \ln(z)$ entonces $e^{p} = e^{\ln(z)} \rightarrow e^{p} = z$
Dicha igualdad completa la sustitución:
$\int e^{p} \operatorname{sen}(p)\, dp$
Ahora, para integrar por partes, se propone:
$u = \operatorname{sen}(p)$ $\quad du = \cos(p)\, dp$
$dv = e^{p} dp$ $\quad v = e^{p}$
$\int u\,dv = uv - \int v\,du$
$e^{p} \operatorname{sen}(p) - \int e^{p} \cos(p)\, dp$
Para integrar por partes, se propone:
$u = \cos(p)$ $\quad du = -\operatorname{sen}(p)\, dp$
$dv = e^{p} dp$ $\quad v = e^{p}$
$\int u\,dv = uv - \int v\,du$
$e^{p} \operatorname{sen}(p) - [e^{p} \cos(p) - \int -e^{p} \operatorname{sen}(p)\, dp]$
$e^{p} \operatorname{sen}(p) - e^{p} \cos(p) - \int e^{p} \operatorname{sen}(p)\, dp$
Por propiedad de transitividad, podemos establecer la igualdad:
$\int e^{p} \operatorname{sen}(p)\, dp = e^{p} \operatorname{sen}(p) - e^{p} \cos(p) - \int e^{p} \operatorname{sen}(p)\, dp$
$2\int e^{p} \operatorname{sen}(p)\, dp = e^{p} \operatorname{sen}(p) - e^{p} \cos(p)$
$$\frac{e^{p} \operatorname{sen}(p) - e^{p} \cos(p)}{2} = \frac{z \operatorname{sen}(\ln z) - z \cos(\ln z)}{2} + c$$

o) $\int w\, [\ln(w)]^2\, dw$
Para integrar por partes, se propone:
$u = [\ln(x)]^2$ $\quad du = \frac{2\ln(x)}{x} dx$
$dv = x\, dx$ $\quad v = \frac{x^2}{2}$
$\int u\,dv = uv - \int v\,du$
$$[\ln(x)]^2 \cdot \frac{x^2}{2} - \int \frac{x^{\cancel{2}}}{\cancel{2}} \cdot \frac{\cancel{2} \ln(x)}{\cancel{x}}\, dx$$
$$\frac{x^2 [\ln(x)]^2}{2} - \int x \ln(x)\, dx$$
Para integrar por partes, se propone:
$u = \ln(x)$ $\quad du = \frac{dx}{x}$
$dv = x\, dx$ $\quad v = \frac{x^2}{2}$
$\int u\,dv = uv - \int v\,du$
$$\frac{x^2 [\ln(x)]^2}{2} - \left[\frac{x^2 \ln(x)}{2} - \int \frac{x^{\cancel{2}}}{2\cancel{x}}\, dx\right]$$
$$\frac{x^2 [\ln(x)]^2}{2} - \frac{x^2 \ln(x)}{2} + \frac{x^2}{4} + c$$

p) $\int_0^{\frac{\pi}{3}} t \tan^2(t)\, dt$

Para integrar por partes, se propone:

$u = t \qquad du = dt$

$dv = \tan^2(t)\, dt \qquad v = \tan(t) - t$

$\int u\, dv = uv - \int v\, du$

$$t\tan(t) - t^2 - \int \tan(t) - t\, dt$$

$$t\tan(t) - t^2 - \int \tan(t)\, dt + \int t\, dt$$

$$t\tan(t) - t^2 + \ln|\cos(t)| + \frac{t^2}{2} \Big|_0^{\frac{\pi}{3}}$$

$$0.572 - 0$$

$$0.572$$

q) $\int_0^1 r\sqrt{1-r}\, dr$

Para integrar por partes, se propone:

$u = r \qquad du = dr$

$dv = \sqrt{1-r} \cdot dr \qquad v = \frac{2(1-r)^{3/2}}{3}$

$\int u\, dv = uv = \int v\, du$

$$\frac{2r(1-r)^{3/2}}{3} - \int \frac{2(1-r)^{3/2}}{3}$$

$$\frac{2r(1-r)^{3/2}}{3} - \frac{2}{3}\left(-\frac{2(1-r)^{5/2}}{5}\right) \Big|_0^1$$

$$\frac{2r(1-r)^{3/2}}{3} + \frac{4(1-r)^{5/2}}{15} \Big|_0^1$$

$$0 - \frac{4}{15}$$

$$-\frac{4}{15}$$

Fecha de Entrega: 20 de mayo del 2019

Tarea 15: Integración por Sustitución Trigonométrica

1.- Evaluar las siguientes integrales:

a) $\int \frac{1}{(25-x^2)^{3/2}} dx$

Se propone la siguiente sustitución:

$\int \frac{1}{(a^2-u^2)^{3/2}} du$

$a^2 = 25 \quad \therefore \quad a = 5$

$u^2 = x^2 \quad \therefore \quad u = x$

Como existe el elemento a^2-u^2, la sustitución apropiada es:

$u = a \text{ sen } \theta$

$u = 5 \text{ sen } \theta \quad \therefore \quad du = 5 \cos\theta \, d\theta$

Así:

$$\int \frac{5\cos\theta \, d\theta}{[25-(5 \text{ sen }\theta)^2]^{3/2}}$$

$$\int \frac{5\cos\theta \, d\theta}{(25-25 \text{ sen}^2\theta)^{3/2}}$$

$$\int \frac{5\cos\theta \, d\theta}{[25(1-\text{sen}^2\theta)]^{3/2}}$$

$$\int \frac{5\cos\theta \, d\theta}{(25)^{3/2}(1-\text{sen}^2\theta)^{3/2}}$$

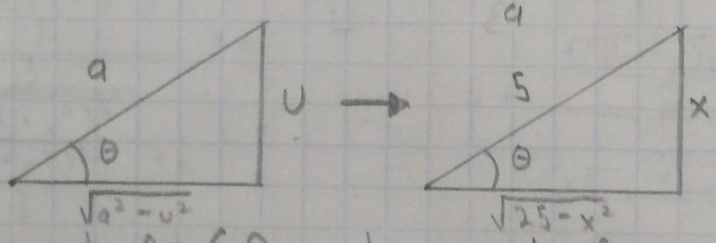
$$\int \frac{5\cos\theta \, d\theta}{125(\cos^2\theta)^{3/2}}$$

$$\int \frac{\cos\theta \, d\theta}{25\cos^3\theta}$$

$$\frac{1}{25}\int \frac{d\theta}{\cos^2\theta} = \frac{1}{25} \cdot \int \sec^2\theta \, d\theta = \frac{\tan\theta}{25} + c$$

Ahora, para deshacer la sustitución trigonométrica, se propone el siguiente triángulo:

Se usó $u = a \text{ sen }\theta \rightarrow \text{sen }\theta = \frac{u}{a}$ y $\text{sen }\theta = \frac{CO}{H}$



Como $\tan\theta = \frac{CO}{CA}$ entonces $\tan\theta = \frac{x}{\sqrt{25-x^2}}$

$$\frac{1}{25} \cdot \frac{x}{\sqrt{25-x^2}} + c$$

$$\frac{x}{25\sqrt{25-x^2}} + c$$

b) $\int z^3 \sqrt{z^2-4}\, dz$

Se propone la siguiente sustitución

$\int u^3 \sqrt{u^2-a^2}\, du$

$a^2 = 4 \quad \therefore \quad a = 2$

$u^2 = z^2 \quad \therefore \quad u = z$

Como existe el elemento u^2-a^2, la sustitución apropiada es:

$u = a \sec\theta$

$u = 2\sec\theta \quad \therefore \quad du = 2\sec\theta \tan\theta\, d\theta$

Así:

$\int (8\sec^3\theta)\sqrt{4\sec^2\theta - 4}\,(2\sec\theta\tan\theta\, d\theta)$

$\int (8\sec^3\theta)\sqrt{4(\sec^2\theta - 1)}\,(2\sec\theta\tan\theta\, d\theta)$

$\int (8)(2)(2)(\sec^3\theta)\sqrt{\tan^2\theta}\,(\sec\theta\tan\theta\, d\theta)$

$32\int (\sec^3\theta)(\tan\theta)(\sec\theta\tan\theta\, d\theta)$

$32\int \sec^4\theta \tan^2\theta\, d\theta$

$32\int \sec^2\theta\tan^2\theta(\tan^2\theta + 1)\, d\theta$

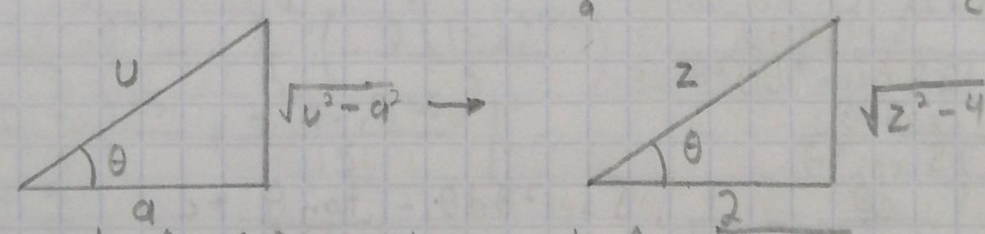
$32\int \sec^2\theta\tan^4\theta + \sec^2\theta\tan^2\theta\, d\theta$

$32\left(\int \sec^2\tan^4\theta\, d\theta + \int \sec^2\tan^2\theta\, d\theta\right)$

$32\left(\frac{\tan^5\theta}{5} + \frac{\tan^3}{3}\right) + c$

Ahora, para deshacer la sustitución trigonométrica, se propone:

Se usó $u = a\sec\theta \rightarrow \sec\theta = \frac{u}{a}$ y $\sec\theta = \frac{H}{CA}$



Como $\tan\theta = \frac{CO}{CA}$ entonces $\tan\theta = \frac{\sqrt{z^2-4}}{2}$

$$32\cdot\left[\frac{\left(\frac{\sqrt{z^2-4}}{2}\right)^5}{5} + \frac{\left(\frac{\sqrt{z^2-4}}{2}\right)^3}{3}\right] + c$$

$$\frac{\frac{32\sqrt{(z^2-4)^5}}{32}}{5} + \frac{\frac{32\sqrt{(z^2-4)^3}}{8}}{3} + c$$

$$\frac{\sqrt{(z^2-4)^5}}{5} + \frac{4\sqrt{(z^2-4)^3}}{3} + c$$

c) $\int x\sqrt{1+x^2}\,dx$

Se propone la siguiente sustitución:

$\int u\sqrt{a^2+u^2}\,du$

$a^2=1 \therefore a=1$

$u^2=x^2 \therefore u=x$

Como existe el elemento a^2+u^2, la sustitución apropiada es:

$u=a\tan\theta$

$u=\tan\theta \therefore du=\sec^2\theta\,d\theta$

Así:

$\int \tan\theta\sqrt{1+\tan^2\theta}\,\sec^2\theta\,d\theta$

$\int \tan\theta\sqrt{\sec^2\theta}\,\sec^2\theta\,d\theta$

$\int \tan\theta\,\sec\theta\,\sec^2\theta\,d\theta$

$\int \tan\theta\,\sec^3\theta\,d\theta$

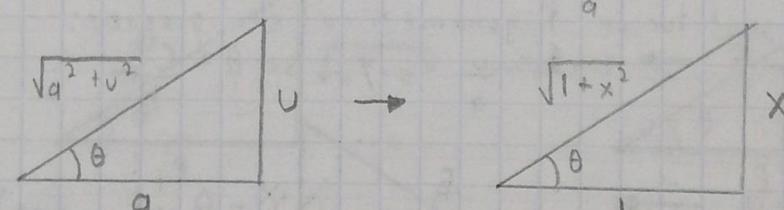
$\int \frac{\operatorname{sen}\theta}{\cos\theta}\cdot\frac{1}{\cos^3\theta}\,d\theta$

$\int \frac{\operatorname{sen}\theta}{\cos^4\theta}\,d\theta$

$\frac{1}{3\cos^3\theta}+c$

Ahora, para deshacer la sustitución trigonométrica se propone:

Se usó $u=a\tan\theta \rightarrow \tan\theta=\frac{u}{a}$ y $\tan\theta=\frac{CO}{CA}$



Como $\cos\theta=\frac{CA}{H}$ entonces $\cos\theta=\frac{1}{\sqrt{1+x^2}}$

$\frac{1}{3\left(\frac{1}{\sqrt{1+x^2}}\right)^3}+c$

$\frac{1}{\frac{3}{\sqrt{(1+x^2)^3}}}+c$

$\frac{\sqrt{(1+x^2)^3}}{3}+c$

d) $\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

Se propone la siguiente sustitución:

$\int \frac{u^2}{\sqrt{a^2-u^2}}\,du$

$a^2 = 25 \quad \therefore \quad a = 5$

$u^2 = x^2 \quad \therefore \quad u = x$

Como existe el elemento a^2-u^2, la sustitución apropiada es:

$u = a \operatorname{sen}\theta$

$u = 5 \operatorname{sen}\theta \qquad du = 5\cos\theta\, d\theta$

Así:

$\int \frac{(5\operatorname{sen}\theta)^2}{\sqrt{25-(5\operatorname{sen}\theta)^2}} \cdot 5\cos\theta\, d\theta$

$\int \frac{(25\operatorname{sen}^2\theta)(5\cos\theta)}{\sqrt{25(1-\operatorname{sen}^2\theta)}}\, d\theta$

$\int \frac{125\operatorname{sen}^2\theta\cos\theta}{5\cos\theta}\, d\theta$

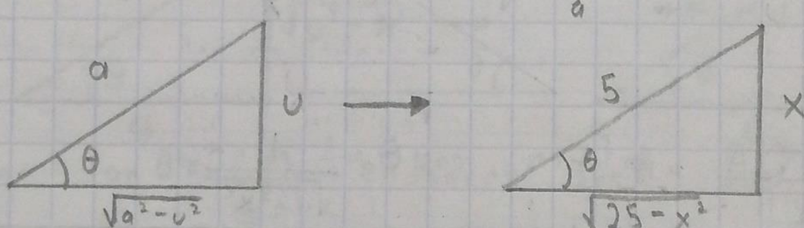
$\int 25\operatorname{sen}^2\theta\, d\theta$

$25\int \operatorname{sen}^2\theta\, d\theta$

$25\left(\frac{\theta}{2} - \frac{1}{4}\operatorname{sen}(2\theta)\right) + c$

Para deshacer la sustitución trigonométrica se propone:

Se usó $u = a\operatorname{sen}\theta \longrightarrow \operatorname{sen}\theta = \frac{u}{a}$ y $\operatorname{sen}\theta = \frac{CO}{H}$



Como $\operatorname{sen}\theta = \frac{CO}{H}$ entonces $\operatorname{sen}\theta = \frac{x}{5}$

$25\left\{\frac{\left[\operatorname{sen}^{-1}\left(\frac{x}{5}\right)\right]}{2} - \frac{1}{4}\cdot 2\operatorname{sen}\theta\cos\theta\right\} + c$

$25\left[\frac{1}{2}\cdot\operatorname{sen}^{-1}\left(\frac{x}{5}\right) - \frac{1}{2}\cdot\frac{x}{5}\cdot\frac{\sqrt{25-x^2}}{5}\right] + c$

$25\left[\frac{1}{2}\cdot\operatorname{sen}^{-1}\left(\frac{x}{5}\right) - \frac{x\sqrt{25-x^2}}{50}\right] + c$

$\frac{25}{2}\cdot\operatorname{sen}^{-1}\left(\frac{x}{5}\right) - \frac{x\sqrt{25-x^2}}{2} + c$

e) $\int \frac{\sqrt{x^2-4}}{x}\,dx$

Se propone la sustitución:

$$\int \frac{\sqrt{u^2-a^2}}{u}\,du$$

$a^2 = 4 \quad \therefore \quad a = 2$

$u^2 = x^2 \quad \therefore \quad u = x$

Como existe el elemento u^2-a^2, la sustitución apropiada es:

$u = a\sec\theta$

$u = 2\sec\theta \qquad du = 2\sec\theta\tan\theta\,d\theta$

Así:

$$\int \frac{\sqrt{4\sec^2\theta-4}}{2\sec\theta}\cdot 2\sec\theta\tan\theta\,d\theta$$

$$\int \frac{\cancel{2}\sqrt{\sec^2\theta-1}}{\cancel{2}\sec\theta}\cdot 2\sec\theta\tan\theta\,d\theta$$

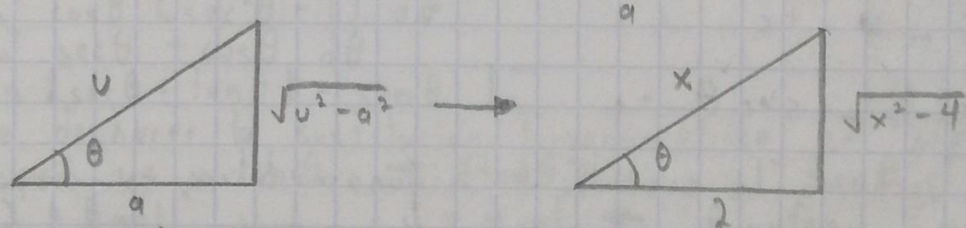
$$\int \tan\theta\cdot 2\tan\theta\,d\theta$$

$$2\int \tan^2\theta\,d\theta$$

$$2\cdot(\tan\theta-\theta)+c$$

Para deshacer la sustitución trigonométrica se propone:

Se usó $u = a\sec\theta \rightarrow \sec\theta = \frac{u}{a}$ y $\sec\theta = \frac{H}{CA}$



Como $\tan\theta = \frac{CO}{CA}$ entonces $\tan\theta = \frac{\sqrt{x^2-4}}{2}$

$$2\left[\frac{\sqrt{x^2-4}}{2} - \cos^{-1}\left(\frac{2}{x}\right)\right]+c$$

$$\frac{\cancel{2}\sqrt{x^2-4}}{\cancel{2}} - 2\cos^{-1}\left(\frac{2}{x}\right)+c$$

$$\sqrt{x^2-4} - 2\cos^{-1}\left(\frac{2}{x}\right)+c$$

f) $\int \frac{y^3}{\sqrt{y^2+1}}\,dy$

Se propone la siguiente sustitución:

$$\int \frac{u^3}{\sqrt{u^2+a^2}}\,du$$

$$a^2 = 1 \quad \therefore \quad a = 1$$

$$u^2 = y^2 \quad \therefore \quad u = y$$

Como existe el elemento u^2+a^2 la sustitución apropiada es:

$$u = a \tan\theta$$

$$u = \tan\theta \qquad du = \sec^2\theta\, d\theta$$

Así:

$$\int \frac{\tan^3\theta}{\sqrt{\tan^2\theta+1}} \cdot \sec^2\theta\, d\theta$$

$$\int \frac{\tan^3\theta}{\sec\theta} \cdot \sec^2\theta\, d\theta$$

$$\int \tan^3\theta \cdot \sec\theta\, d\theta$$

$$\int \tan^2\theta \cdot \tan\theta \cdot \sec\theta\, d\theta$$

$$\int (\sec^2\theta - 1) \cdot \tan\theta \cdot \sec\theta\, d\theta$$

$$v = \sec\theta \qquad dv = \sec\theta \tan\theta\, d\theta$$

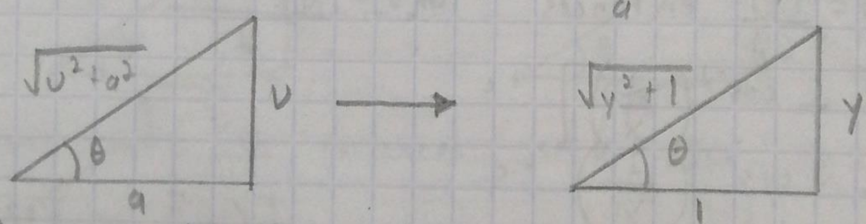
$$\int v^2 - 1\, dv$$

$$\frac{v^3}{3} - v + c$$

$$\frac{\sec^3\theta}{3} - \sec\theta + c$$

Para deshacer la sustitución trigonométrica, se propone:

Se usó $u = a\tan\theta \rightarrow \tan\theta = \frac{u}{a}$ y $\tan\theta = \frac{CO}{CA}$



Como $\sec\theta = \frac{H}{CA}$ entonces $\sec\theta = \frac{\sqrt{y^2+1}}{1}$

$$\frac{\sqrt{(y^2+1)^3}}{3} - \sqrt{y^2+1} + c$$

g) $\int_0^{\frac{\sqrt{3}}{2}} \frac{z^2}{(1+z^2)^{3/2}}\,dz$

Se propone la siguiente sustitución:

$\int_n^m \frac{u^2}{(a^2+u^2)^{3/2}}\,du$

$a^2 = 1 \quad \therefore \quad a = 1$

$u^2 = z^2 \quad \therefore \quad u = z$

Como existe el elemento $a^2 + u^2$ la sustitución apropiada es:

$u = a \tan\theta$

$u = \tan\theta \qquad du = \sec^2\theta\, d\theta$

Así:

$\int_n^m \frac{\tan^2\theta}{(1+\tan^2\theta)^{3/2}} \cdot \sec^2\theta\, d\theta$

$\int_n^m \frac{\tan^2\theta \cdot \sec^2\theta}{(\sec^2\theta)^{3/2}}\, d\theta$

$\int_n^m \frac{\tan^2\theta \sec^2\theta}{\sec^3\theta}\, d\theta$

$\int_n^m \frac{\tan^2\theta}{\sec\theta}\, d\theta$

$\int_n^m \cos\theta \tan^2\theta\, d\theta$

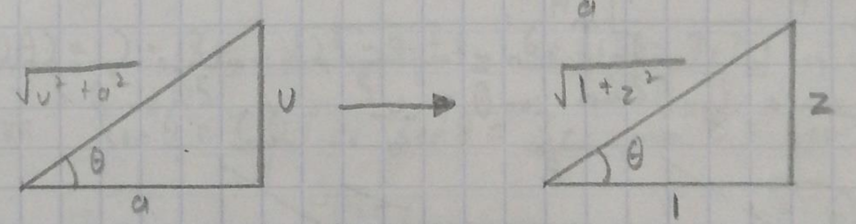
$\int_n^m \cos\theta\,(\sec^2\theta - 1)\, d\theta$

$\int_n^m \sec\theta - \cos\theta\, d\theta$

$\ln|\sec\theta + \tan\theta| - \operatorname{sen}\theta \Big|_n^m$

Para deshacer la sustitución trigonométrica se propone

Se usó $u = a\tan\theta \rightarrow \tan\theta = \frac{u}{a}$ y $\tan\theta = \frac{CO}{CA}$



$\sec\theta = \frac{H}{CA} \rightarrow \sec\theta = \sqrt{1+z^2}, \quad \tan\theta = z, \quad \operatorname{sen}\theta = \frac{z}{\sqrt{1+z^2}}$

$\ln\left|\sqrt{1+z^2} + z\right| - \frac{z}{\sqrt{1+z^2}} \Big|_0^{\frac{\sqrt{3}}{2}}$

$0.129 - 0$

0.129

h) $\int_0^3 \frac{y^3}{\sqrt{y^2+9}}\, dy$

Se propone la siguiente sustitución

$$\int \frac{u^3}{\sqrt{a^2+u^2}}\, du$$

$$a^2 = 9 \quad \therefore \quad a = 3$$
$$u^2 = y^2 \quad \therefore \quad u = y$$

Como existe el elemento $u^2 + a^2$ la sustitución apropiada es:

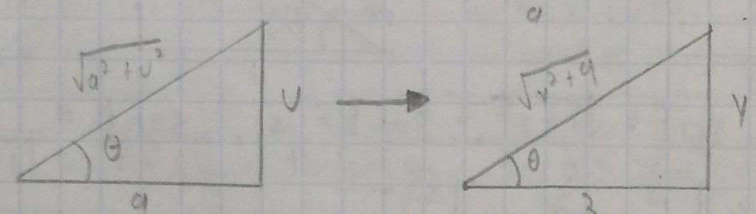
$$u = a \tan\theta$$
$$u = 3\tan\theta \qquad du = 3\sec^2\theta\, d\theta$$

Así:

$$\int_m^n \frac{27\tan^3\theta}{\sqrt{9+9\tan^2\theta}} \cdot 3\sec^2\theta\, d\theta$$
$$\int_m^n \frac{27\tan^3\theta}{3\sqrt{1+\tan^2\theta}} \cdot 3\sec^2\theta\, d\theta$$
$$\int_m^n \frac{27\tan^3\theta\sec^2\theta}{\sec\theta}\, d\theta$$
$$27\int_m^n \tan^3\theta\sec\theta\, d\theta$$
$$27\int_m^n \tan\theta\cdot\tan^2\theta\sec\theta\, d\theta$$
$$27\int_m^n \tan\theta(\sec^2-1)\sec\theta\, d\theta$$
$$v = \sec\theta \qquad dv = \sec\theta\tan\theta\, d\theta$$
$$27\int_{m'}^{n'} (v^2-1)\, dv$$
$$27\left[\frac{v^3}{3} - v\right]\Big|_{m'}^{n'}$$
$$27\left[\frac{\sec^3\theta}{3} - \sec\theta\right]_m^n$$

Para deshacer la sustitución trigonométrica se propone:

Se usó $u = a\tan\theta \rightarrow \tan\theta = \frac{u}{a}$ y $\tan\theta = \frac{CO}{CA}$



Como $\sec\theta = \frac{H}{CA}$ entonces $\sec\theta = \frac{\sqrt{y^2+9}}{3}$

$$\frac{\sqrt{(y^2+9)^3}}{3} - 9\sqrt{y^2+9}\ \Big|_0^3$$
$$-9\sqrt{2} - (-18)$$
$$-9\sqrt{2} + 18$$
$$18 - 9\sqrt{2}$$
$$5.272$$

i) $\int_0^{5/3} \sqrt{25-9r^2}\,dr$

Se propone la sustitución:

$\int \sqrt{a^2-u^2}\,du$

$a^2 = 25 \quad \therefore a = 5$

$u^2 = 9r^2 \quad \therefore u = 3r$

Como existe a^2-u^2 la sustitución apropiada es:

$u = a\,\text{sen}\,\theta$

$u = 5\,\text{sen}\,\theta \qquad du = 5\cos\theta\,d\theta$

Así:

$\int_m^n \sqrt{25-25\,\text{sen}^2\theta}\cdot 5\cos\theta\,d\theta$

$\int_m^n \sqrt{25}\,\sqrt{1-\text{sen}^2\theta}\cdot 5\cos\theta\,d\theta$

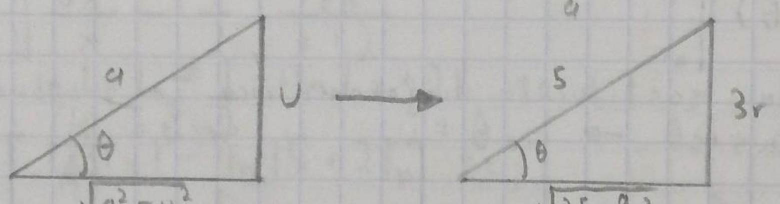
$\int_m^n 5\cdot\cos\theta\cdot 5\cos\theta\,d\theta$

$25\int_m^n \cos^2\theta\,d\theta$

$25\cdot\left(\frac{\theta}{2}+\frac{1}{4}\cdot\text{sen}(2\theta)\right)\Big|_m^n$

Para deshacer la sustitución trigonométrica se propone:

Se usó $u = a\,\text{sen}\,\theta \rightarrow \text{sen}\,\theta = \frac{u}{a}$ y $\text{sen}\,\theta = \frac{CO}{H}$



$\text{sen}(2\theta) = 2\,\text{sen}\,\theta\cos\theta$, si $\text{sen}\,\theta = \frac{CO}{H}$ y $\cos\theta = \frac{CA}{H}$ entonces:

$\text{sen}(2\theta) = 2\cdot\frac{3r}{5}\cdot\frac{\sqrt{25-9r^2}}{5} = \frac{6r\sqrt{25-9r^2}}{25}$

También $\text{sen}\,\theta = \frac{CO}{H} \therefore \text{sen}\,\theta = \frac{3r}{5} \rightarrow \theta = \text{sen}^{-1}\left(\frac{3r}{5}\right)$

Así:

$$25\cdot\left[\frac{\text{sen}^{-1}\left(\frac{3r}{5}\right)}{2}+\frac{\left(\frac{6r\sqrt{25-9r^2}}{25}\right)}{4}\right]\Bigg|_0^{5/3}$$

$$\frac{25\,\text{sen}^{-1}\left(\frac{3r}{5}\right)}{2}+\frac{3r\sqrt{25-9r^2}}{2}\Bigg|_0^{5/3}$$

$$\frac{25\,\text{sen}^{-1}\left(\frac{3r}{5}\right)+3r\sqrt{25-9r^2}}{2}\Bigg|_0^{5/3}$$

$$\frac{25\pi}{4}-0=\frac{25\pi}{4}$$

j) $\int_{\sqrt{2}}^{2} \frac{1}{x^3\sqrt{x^2-1}}\,dx$

Se propone la siguiente sustitución

$$\int \frac{1}{u^3\sqrt{u^2-a^2}}\,dx$$

$$a^2=1 \quad \therefore \quad a=1$$
$$u^2=x^2 \quad \therefore \quad u=x$$

Como existe u^2-a^2 la sustitución apropiada es:

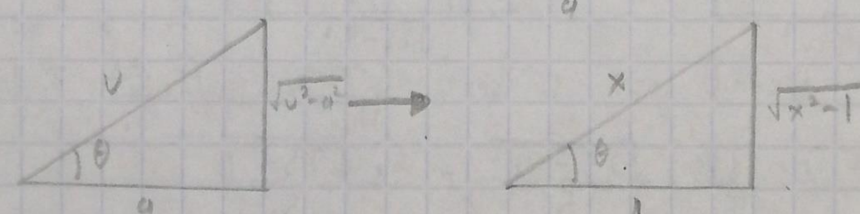
$$u=a\sec\theta$$
$$u=\sec\theta \qquad du=\sec\theta\tan\theta\,d\theta$$

Así

$$\int_m^n \frac{1}{\sec^3\theta\sqrt{\sec^2\theta-1}}\cdot\sec\theta\tan\theta\,d\theta$$
$$\int_m^n \frac{\sec\theta\tan\theta}{\sec^3\theta\tan\theta}\,d\theta$$
$$\int_m^n \frac{1}{\sec^2\theta}\,d\theta$$
$$\int_m^n \cos^2\theta\,d\theta$$
$$\frac{\theta}{2}+\frac{1}{4}\cdot sen(2\theta)\Big|_m^n$$

Para deshacer la sustitución trigonométrica se propone

Se uso $u=a\sec\theta \rightarrow \sec\theta=\frac{u}{a}$ y $\sec\theta=\frac{H}{CA}$



$$sen(2\theta)=2\,sen\theta\cos\theta=2\cdot\frac{\sqrt{x^2-1}}{x}\cdot\frac{1}{x}=\frac{2\sqrt{x^2-1}}{x^2}$$

$$\tan\theta=\sqrt{x^2-1} \quad \therefore \quad \theta=\tan^{-1}\left(\sqrt{x^2-1}\right)$$

Así:

$$\frac{\tan^{-1}\left(\sqrt{x^2-1}\right)}{2}+\frac{\sqrt{x^2-1}}{2x^2}\Bigg|_{\sqrt{2}}^{2}$$

Tras analizar la función, existe una discontinuidad en $x=\sqrt{2}$
Sin embargo, aplicando un análisis de integrales impropias se obtiene después de la evaluación del límite:

0.740 − 0.643

0.0974

Fecha de Entrega: 10 de junio del 2019

Tarea 16: Integración por fracciones parciales (primera parte)

1.- Calcular las siguientes integrales:

a) $\int \frac{1}{x^2+2x}\,dx$

Se propone la siguiente separación en fracciones simples:

$$\frac{1}{x^2+2x} = \frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

$$\frac{Ax+2A+Bx}{x(x+2)} = \frac{(A+B)x+2A}{x(x+2)}$$

$$\frac{0x+1}{x^2+2x} = \frac{(A+B)x+2A}{x^2+2x}$$

$$A+B=0$$
$$2A=1 \quad \therefore A=\tfrac{1}{2} \quad \therefore B=-\tfrac{1}{2}$$

$$\frac{1}{2x} - \frac{1}{2(x+2)}$$

$$\int \left[\frac{1}{2x} - \frac{1}{2(2+x)}\right] dx$$

$$\frac{1}{2}\int \frac{dx}{x} - \frac{1}{2}\int \frac{dx}{x+2}$$

$$\frac{1}{2}\cdot \ln|x| - \frac{1}{2}\cdot \ln|x+2| + c$$

$$\frac{1}{2}\left[\ln|x| - \ln|x+2|\right] + c$$

$$\frac{1}{2}\left[\ln\left(\frac{x}{x+2}\right)\right] + c$$

$$\ln\sqrt{\frac{x}{x+2}} + c$$

b) $\int \frac{z+4}{z^2+5z-6}\,dz$

Se propone la siguiente separación en fracciones simples:

$$\frac{z+4}{z^2+5z-6} = \frac{z+4}{(z+6)(z-1)} = \frac{A}{z+6} + \frac{B}{z-1}$$

$$\frac{Az-A+Bz+6B}{z^2+5z-6} = \frac{(A+B)z+(6B-A)}{z^2+5z-6}$$

$$\frac{z+4}{z^2+5z-6} = \frac{(A+B)z+(6B-A)}{z^2+5z-6}$$

$$A+B=1$$
$$6B-A=4 \longrightarrow 6B-4=A \quad \therefore 6B-4+B=1 \quad \therefore B=\tfrac{5}{7}$$
$$\therefore A=\tfrac{2}{7}$$

$$\frac{2}{7(z+6)} + \frac{5}{7(z-1)}$$

$$\int \left[\frac{2}{7(z+6)} + \frac{5}{7(z-1)} \right] dz$$

$$\frac{2}{7}\int \frac{dz}{z+6} + \frac{5}{7}\int \frac{dz}{z-1}$$

$$\frac{2}{7}\cdot \ln|z+6| + \frac{5}{7}\cdot \ln|z-1| + c$$

$$\ln\left|\sqrt[7]{(z+6)^2(z-1)^5}\right| + c$$

c) $\displaystyle\int \frac{2x+1}{x^2-7x+12}\,dx$

Se propone la siguiente separación en fracciones simples:

$$\frac{2x+1}{x^2-7x+12} = \frac{2x+1}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$$

$$\frac{Ax-3A+Bx-4B}{x^2-7x+12} = \frac{(A+B)x+(-3A-4B)}{x^2-7x+12}$$

$$A+B=2$$

$$-3A-4B=1 \Rightarrow -3(2-B)-4B=1 \rightarrow -6+3B-4B=1$$

$$\therefore B=-7 \text{ y } A=9$$

$$\frac{9}{x-4} - \frac{7}{x-3}$$

$$9\int \frac{dx}{x-4} - 7\int \frac{dx}{x-3}$$

$$9\cdot \ln|x-4| - 7\cdot \ln|x-3| + c$$

$$\ln\left|\frac{(x-4)^9}{(x-3)^7}\right| + c$$

d) $\displaystyle\int_4^8 \frac{y}{y^2-2y-3}\,dy$

Se propone la separación:

$$\frac{y}{y^2-2y-3} = \frac{y}{(y-3)(y+1)} = \frac{A}{y-3} + \frac{B}{y+1}$$

$$\frac{Ay+A+By-3B}{y^2-2y-3} = \frac{(A+B)y+(A-3B)}{y^2-2y-3}$$

$$A+B=1$$

$$A-3B=0 \Rightarrow 1-B-3B=0 \quad \therefore B=\tfrac{1}{4} \quad \therefore A=\tfrac{3}{4}$$

$$\frac{1}{4(y-3)} + \frac{3}{4(y+1)}$$

$$\frac{1}{4}\int \frac{dy}{y-3} + \frac{3}{4}\int \frac{dy}{y+1}$$

$$\frac{1}{4}\cdot \ln|y-3| + \frac{3}{4}\cdot \ln|y+1| \Big|_4^8$$

$$\ln\left|\sqrt[4]{(y-3)(y+1)^3}\right| \Big|_4^8$$

$$4.101 - 2.414 = 1.686$$

e) $\int_{1/2}^{1} \frac{x+4}{x^2+x}\,dx$

Se propone la siguiente separación:

$$\frac{x+4}{x^2+x} = \frac{x+4}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$\frac{Ax + A + Bx}{x(x+1)} = \frac{(A+B)x + A}{x(x+1)}$$

$$A + B = 1$$
$$A = 4 \qquad \therefore \quad B = -3$$

$$\frac{4}{x} - \frac{3}{x+1}$$

$$4\int_{1/2}^{1} \frac{dx}{x} - 3\int_{1/2}^{1} \frac{dx}{x+1}$$

$$4 \cdot \ln|x| - 3 \cdot \ln|x+1| \Big|_{1/2}^{1}$$

$$\ln\left|\frac{x^4}{(x+1)^3}\right| \Bigg|_{1/2}^{1}$$

$$-2.079 - (-3.989)$$
$$1.909$$

f) $\int \frac{1}{(y^2-1)^2}\,dy$

Se propone la separación:

$$\frac{1}{(y^2-1)^2} = \frac{A}{(y-1)} + \frac{B}{(y+1)} + \frac{C}{(y-1)^2} + \frac{D}{(y+1)^2}$$

$$\frac{Ay^3 + Ay^2 - Ay - A + By^3 - By^2 - By + B + Cy^2 + 2Cy + C + Dy^2 - 2Dy + D}{(y^2-1)^2}$$

$$\frac{(A+B)y^3 + (A-B+C+D)y^2 + (-A-B+2C-2D)y + (-A+B+C+D)}{(y^2-1)^2}$$

$$A + B = 0$$
$$A - B + C + D = 0$$
$$-A - B + 2C - 2D = 0$$
$$-A + B + C + D = 1$$

De aquí: $A = -\frac{1}{4}$, $B = \frac{1}{4}$, $C = \frac{1}{4}$, $D = \frac{1}{4}$

$$-\frac{1}{4(y-1)} + \frac{1}{4(y+1)} + \frac{1}{4(y-1)^2} + \frac{1}{4(y+1)^2}$$

$$-\frac{1}{4}\int \frac{dy}{y-1} + \frac{1}{4}\int \frac{dy}{y+1} + \frac{1}{4}\int \frac{dy}{(y-1)^2} + \frac{1}{4}\int \frac{dy}{(y+1)^2}$$

$$\frac{1}{4}\left[\ln|y+1| - \ln|y-1| - \frac{1}{y-1} - \frac{1}{y+1}\right] + c$$

$$\ln\left|\sqrt[4]{\frac{y+1}{y-1}}\right| - \frac{1}{4y-4} - \frac{1}{4y+4} + c$$

g) $\int \frac{t^2+1}{(t-3)(t-2)^2}\,dt$

Se propone:

$$\frac{A}{(t-3)}+\frac{B}{(t-2)}+\frac{C}{(t-2)^2}$$

$$\frac{At^2-4At+4A+Bt^2-5Bt+6B+Ct-3C}{(t-3)(t-2)^2}$$

$$\frac{(A+B)t^2+(-4A-5B+C)t+(4A+6B-3C)}{(t-3)(t-2)^2}$$

$A+B=1$ De aquí: $A=10$

$-4A-5B+C=0$ $B=-9$

$4A+6B-3C=1$ $C=-5$

$$\frac{10}{(t-3)}-\frac{9}{(t-2)}-\frac{5}{(t-2)^2}$$

$$10\int\frac{dt}{t-3}-9\int\frac{dt}{t-2}-5\int\frac{dt}{(t-2)^2}$$

$$10\cdot\ln|t-3|-9\cdot\ln|t-2|+\frac{5}{t-2}+c$$

h) $\int \frac{z^2}{(z-1)(z^2+2z+1)}\,dz$

Se propone:

$$\frac{A}{z-1}+\frac{B}{z+1}+\frac{C}{(z+1)^2}$$

$$\frac{Az^2+2Az+A+Bz^2-B+Cz-C}{(z-1)(z^2+2z+1)}$$

$$\frac{(A+B)z^2+(2A+C)z+(A-B-C)}{(z-1)(z^2+2z+1)}$$

$A+B=1$ Así: $A=\frac{1}{4}$

$2A+C=0$ $B=\frac{3}{4}$

$A-B-C=0$ $C=-\frac{1}{2}$

$$\frac{1}{4(z-1)}+\frac{3}{4(z+1)}-\frac{1}{2(z+1)^2}$$

$$\frac{1}{4}\int\frac{dz}{z-1}+\frac{3}{4}\int\frac{dz}{z+1}-\frac{1}{2}\int\frac{dz}{(z+1)^2}$$

$$\frac{1}{4}\cdot\ln|z-1|+\frac{3}{4}\cdot\ln|z+1|+\frac{1}{2(z+1)}+c$$

$$\ln\left(\sqrt[4]{(z-1)(z+1)^3}\right)+\frac{1}{2(z+1)}+c$$

i) $\int_0^1 \frac{r^3}{r^2+2r+1}\,dr$

Se propone

$$\frac{A}{r+1}+\frac{B}{(r+1)^2}=\frac{Ar+A+B}{(r+1)^2}$$

$A=0$

$A+B=0 \quad \therefore \quad B=0$

Con los coeficientes obtenidos, la fracción es 0, por lo que la propuesta no cumple y no se satisface

$$\frac{r^3+2r^2+r-2r^2-r}{r^2+2r+1}=\frac{r\cancel{(r^2+2r+1)}}{\cancel{r^2+2r+1}}-\frac{2r^2+r}{r^2+2r+1}$$

$$r-\frac{2r^2+r+3r-3r+2-2}{r^2+2r+1}=r-\frac{2r^2+4r+2-3r-2}{r^2+2r+1}$$

$$r-\frac{2\cancel{(r^2+2r+1)}}{\cancel{r^2+2r+1}}-\frac{3r+2}{r^2+2r+1}$$

Para el último término, se propone nuevamente la separación en fracciones simples, pero ahora:

$A=3$

$A+B=2 \quad \therefore \quad B=-1$

$$\frac{3}{r+1}-\frac{1}{(r+1)^2}$$

$$\int_0^1 r-2-\frac{3}{r+1}+\frac{1}{(r+1)^2}\,dr$$

$$\frac{r^2}{2}-2r-3\cdot\ln|r+1|-\frac{1}{r+1}\Big|_0^1$$

$-4.079-(-1)$

-3.079

j) $\int_{-1}^0 \frac{y^3}{y^2-2y+1}\,dy$

Se propone

$$\frac{A}{(y-1)}+\frac{B}{(y-1)^2}=\frac{Ay-A+B}{(y-1)^2}$$

$A=0$

$-A+B=0 \quad \therefore \quad B=0$

La fracción es 0 y la propuesta no se satisface

$$\frac{y^3-2y^2+y+2y^2-y}{y^2-2y+1}=\frac{y\cancel{(y^2-2y+1)}}{\cancel{y^2-2y+1}}+\frac{2y^2-y}{y^2-2y+1}$$

$$y+\frac{2y^2-y-3y+2+3y-2}{y^2-2y+1}=y+\frac{2\cancel{(y^2-2y+1)}}{\cancel{y^2-2y+1}}+\frac{3y-2}{y^2-2y+1}$$

$$y+2+\frac{3y-2}{y^2-2y+1}$$

Para el último término se propone la separación, pero ahora:

$$A = 3$$
$$-A + B = -2 \quad \therefore \quad B = 5$$

$$\frac{3}{y-1} + \frac{5}{(y-1)^2}$$

$$\int_{-1}^{0} y + 2 + \frac{3}{y-1} + \frac{5}{(y-1)^2} \, dy$$

$$\frac{y^2}{2} + 2y + 3\ln|y-1| - \frac{5}{y-1} \Big|_{-1}^{0}$$

$$5 - 2.079$$
$$2.921$$

Fecha de Entrega: 10 de junio del 2019

Tarea 17: Integración por fracciones parciales (Segunda parte)

1.- Calcular las siguientes integrales

a) $\int \frac{x^2-1}{x^3-x}\,dx$

Se propone:

$\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}=\frac{Ax^2-A+Bx^2-Bx+Cx^2+Cx}{x^3-x}$

$\frac{(A+B+C)x^2+(-B+C)x+(-A)}{x^3-x}$

$A+B+C=1$
$-B+C=0$
$-A=-1 \quad \therefore \quad A=1 \quad \therefore \quad B=0 \quad , \quad C=0$

$\frac{1}{x}$

$\int \frac{dx}{x}$

$\ln|x|+c$

b) $\int \frac{z}{z^3-1}\,dz$

Se propone:

$\frac{A}{z-1}+\frac{Bz+C}{z^2+z+1}=\frac{Az^2+Az+A+Bz^2-Bz+Cz-C}{z^3-1}$

$\frac{(A+B)z^2+(A-B+C)z+(A-C)}{z^3-1}$

$A+B=0$
$A-B+C=1$
$A-C=0$

Por método de Kramer:

$$\begin{matrix}1 & 1 & 0 & 0\\ 1 & -1 & 1 & 1\\ 1 & 0 & -1 & 0\end{matrix}$$

$$\begin{vmatrix}1 & 1 & 0\\ 1 & -1 & 1\\ 1 & 0 & -1\end{vmatrix}\quad \begin{matrix}1 & 1 & 0\\ 1 & -1 & 1\end{matrix}$$

$[1+0+1]-[-1+0+0]$
$[2]-[-1]$
3

Para resolver A

$$\begin{vmatrix}0 & 1 & 0\\ 1 & -1 & 1\\ 0 & 0 & -1\end{vmatrix}\quad \begin{matrix}0 & 1 & 0\\ 1 & -1 & 1\end{matrix}$$

$[0+0+0]-[-1+0\,0]$
1

$A = \frac{1}{3}$

$\therefore B = -\frac{1}{3} \qquad \therefore C = \frac{1}{3}$

$$\frac{1}{3(z-1)} + \frac{-\frac{1}{3}\cdot z + \frac{1}{3}}{z^2+z+1}$$

$$\frac{1}{3}\int \frac{dz}{z-1} + \frac{1}{3}\int \frac{-z+1}{z^2+z+1}\,dz$$

$$\frac{1}{3}\int \frac{dz}{z-1} - \frac{1}{3}\int \frac{z-1}{z^2+z+1}\,dz$$

$$\frac{1}{3}\int \frac{dz}{z-1} - \frac{1}{6}\int \frac{2z-2}{z^2+z+1}\,dz$$

$$\frac{1}{3}\int \frac{dz}{z-1} - \frac{1}{6}\int \frac{2z+1-3}{z^2+z+1}\,dz$$

$$\frac{1}{3}\int \frac{dz}{z-1} - \frac{1}{6}\int \frac{2z+1}{z^2+z+1}\,dz + \frac{1}{6}\int \frac{3}{z^2+z+1}\,dz$$

$$\frac{1}{3}\int \frac{dz}{z-1} - \frac{1}{6}\int \frac{2z+1}{z^2+z+1}\,dz + \frac{1}{2}\int \frac{1}{(z+\frac{1}{2})^2+\frac{3}{4}}\,dz$$

$$\frac{1}{3}\cdot \ln|z-1| - \frac{1}{6}\cdot \ln|z^2+z+1| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2z+1}{\sqrt{3}}\right) + k$$

c) $\displaystyle\int \frac{x^2}{x^4-2x^2-8}\,dx$

Se propone:

$$\frac{x^2}{(x^2-4)(x^2+2)} = \frac{x^2}{(x-2)(x+2)(x^2+2)}$$

$$\frac{A}{x-2} + \frac{B}{x+2} + \frac{Cx+D}{x^2+2}$$

$$\frac{Ax^3+2Ax^2+2Ax+4A+Bx^3-2Bx^2+2Bx-4B+Cx^3-4Cx+Dx^2-4D}{x^4-2x^2-8}$$

$$\frac{(A+B+C)x^3+(2A-2B+D)x^2+(2A+2B-4C)x+(4A-4B-4D)}{x^4-2x^2-8}$$

$A+B+C=0$
$2A-2B+D=1$
$2A+2B-4C=0$
$4A-4B-4D=0$

De ahí: $A = \frac{1}{6}$ $\quad C = 0$ $\quad B = -\frac{1}{6}$ $\quad D = \frac{1}{3}$

$$\frac{1}{6(x-2)} - \frac{1}{6(x+2)} + \frac{-\frac{2}{3}x+\frac{1}{3}}{x^2+2}$$

$$\frac{1}{6}\int \frac{dx}{x-2} - \frac{1}{6}\int \frac{dx}{x+2} + \int \frac{-\ \ x + 1/3}{x^2+2}\,dx$$

$$\frac{1}{6}\int \frac{dx}{x-2} - \frac{1}{6}\int \frac{dx}{x+2} - \qquad \frac{1}{3}\int \frac{1}{x^2+2}\,dx$$

$\frac{1}{2} \cdot \ln|x-2| + \frac{1}{6} \cdot \ln|x+2| \cdot \ln|x^2+2| + \frac{1}{3\sqrt{2}} \cdot \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

d) $\int \frac{r}{16r^4-1}\,dr$

Se propone:

$\frac{r}{(4r^2-1)(4r^2+1)} = \frac{rA}{(2r-1)(2r+1)(4r^2+1)}$

$\frac{A}{2r-1} + \frac{B}{2r+1} + \frac{Cr+D}{4r^2+1}$

$\frac{8Ar^3 + 4Ar^2 + 2Ar + A + 8Br^3 - 4Br^2 + 2Br - B + 4Cr^3 - Cr + 4Dr^2 - D}{16r^4-1}$

$\frac{(8A+8B+4C+4D)r^3 + (4A-4B)r^2 + (2A+2B)r + (A-B-C-D)}{16r^4-1}$

$8A+8B+4C=0$

$4A-4B+4D=0$

$2A+2B-C=1$

$A-B-D=0$

Así: $A=\frac{1}{8}$, $B=\frac{1}{8}$, $C=-\frac{1}{2}$, $D=0$

$\frac{1}{8(2r-1)} + \frac{1}{8(2r+1)} - \frac{r}{2(4r^2+1)}$

$\frac{1}{8} \cdot \ln|2r-1| + \frac{1}{8} \cdot \ln|2r+1| - \frac{1}{16} \cdot \ln|4r^2+1| + c$

e) $\int \frac{t^2-t+6}{t^3+3t}\,dt$

Se propone:

$t^3+3t = t(t^2+3)$

$\frac{A}{t} + \frac{Bt+C}{t^2+3} = \frac{At^2+3A+Bt^2+Ct}{t^3+3t} = \frac{(A+B)t^2+(C)t+(3A)}{t^3+3t}$

$A+B=1$

$C=-1$

$3A=6$

$A=2,\ B=-1,\ C=-1$

$\frac{2}{t} + \frac{-1t-1}{t^2+3}$

$2 \cdot \ln|t| - \frac{1}{2} \cdot \ln|t^2+3| - \frac{1}{\sqrt{3}} \cdot \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + c$

f) $\int \frac{y^2+y+1}{y^4+2y^2+1}\,dy$

Se propone

$\frac{y^2+y+1}{(y^2+1)^2} \rightarrow \frac{Ay+B}{y^2+1} + \frac{Cy+D}{(y^2+1)^2} = \frac{Ay^3+Ay+By^2+B+Cy+D}{(y^2+1)^2}$

$Az^3 + 2Az + Bz^2 + 2B$

$$\frac{(A)y^3 + (B)y^2 + (A+C)y + (B+D)}{(y^2+1)^2}$$

$A = 0$
$B = 1$
$A + C = 1 \quad \therefore \quad C = 1$
$B + D = 1 \quad \therefore \quad D = 0$

$$\frac{1}{y^2+1} + \frac{y}{(y^2+1)^2}$$

$$\tan^{-1}(y) - \frac{1}{2} \cdot \frac{1}{y^2+1} + c$$

$$\tan^{-1}(y) - \frac{1}{2y^2+2} + c$$

g) $\int \frac{z^3 + z^2 + 2z + 1}{z^4 + 4z^2 + 4}\, dz$

Se propone:

$$\frac{z^3+z^2+2z+1}{(z^2+2)^2} \rightarrow \frac{Az+B}{z^2+2} + \frac{Cz+D}{(z^2+2)^2} = \frac{Az^3 + 2Az + Bz^2 + 2B + Cz + D}{(z^2+2)^2}$$

$$\frac{(A)z^3 + (B)z^2 + (2A+C)z + (2B+D)}{(z^2+2)^2}$$

$A = 1$
$B = 1$
$2A + C = 2 \quad \therefore \quad C = 0$
$2B + D = 1 \quad \therefore \quad D = -1$

$$\frac{z+1}{z^2+2} - \frac{1}{(z^2+2)^2} \Rightarrow ?$$

$$\frac{1}{2} \cdot \ln|z^2+2| + \frac{1}{\sqrt{2}} \cdot \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \dots + c$$

Fecha de Entrega: 17 de junio del 2019

Tarea 18: Series de Taylor y Maclaurin (Primera Parte)

1 - Probar que

a)

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

La serie de Maclaurin se define:

$$f(0) + f'(0) + \frac{f''(0) \cdot x^2}{2!} + \frac{f'''(0)\, x^3}{3!} + \dots + \frac{f^{(n)}\, x^n}{n!}$$

Derivando e^x

$f(x) = e^x$, $f'(x) = e^x$, $f''(x) = e^x$, $f'''(x) = e^x \dots$

Así:

$$\frac{e^0 \cdot x^0}{0!} + \frac{e^0 \cdot x^1}{1!} + \frac{e^0 \cdot x^2}{2!} + \frac{e^0 \cdot x^3}{3!} + \frac{e^0 \cdot x^4}{4!} + \dots$$

$$\frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \quad \frac{x^k}{k!}$$

Se reescribe:

$$\sum_{k=0}^{\infty} \frac{x^k}{k!}$$

b)

$$\cos(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \cdot x^{2k}$$

Derivando $\cos(x)$

$f(x) = \cos(x)$, $f'(x) = -\text{sen}(x)$, $f''(x) = -\cos(x)$, $f'''(x) = \text{sen}(x)$,
$f^{(4)}(x) = \cos(x)$

Así:

$$\frac{\cos(0) \cdot x^0}{0!} + \frac{-\text{sen}(0) \cdot x^1}{1!} + \frac{-\cos(0)\, x^2}{2!} + \frac{\text{sen}(0)\, x^3}{3!} + \frac{\cos(0)\, x^4}{4!} + \dots$$

$$\frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \quad (-1)^k \cdot \frac{x^{2k}}{(2k)!}$$

Se reescribe:

$$\sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k}}{(2k)!}$$

c)

$$\text{sen}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \cdot x^{2k+1}$$

Derivando $\text{sen}(x)$

$f(x) = \text{sen}(x)$, $f'(x) = \cos(x)$, $f''(x) = -\text{sen}(x)$, $f'''(x) = -\cos(x)$, $f^{(4)}(x) = \text{sen}(x)$

Así:

$$\frac{\text{sen}(0) \cdot x^0}{0!} + \frac{\cos(0) \cdot x^1}{1!} + \frac{-\text{sen}(0) \cdot x^2}{2!} + \frac{-\cos(0)\, x^3}{3!} + \frac{\text{sen}(0) \cdot x^4}{4!} \dots$$

$$\frac{x^1}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \quad (-1)^k \cdot \frac{x^{2k+1}}{(2k+1)!}$$

Se reescribe:

$$\sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k+1}}{(2k+1)!}$$

d)

$$\operatorname{senh}(x) = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$$

Derivando $\operatorname{senh}(x)$

$f(x) = \operatorname{senh}(x)$, $f'(x) = \cosh(x)$, $f''(x) = \operatorname{senh}(x)$, $f'''(x) = \cosh(x)$, $f^{(4)}(x) = \operatorname{senh}(x)$

Así:
$$\frac{\operatorname{senh}(0) \cdot x^0}{0!} + \frac{\cosh(0) \cdot x^1}{1!} + \frac{\operatorname{senh}(0) \cdot x^2}{2!} + \frac{\cosh(0) \cdot x^3}{3!} + \frac{\operatorname{senh}(0) \cdot x^4}{4!} \dots$$

$$\frac{x^1}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \quad \frac{x^{2k+1}}{(2k+1)!}$$

Se reescribe:

$$\sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$$

e)

$$\cosh(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

Derivando $\cosh(x)$

$f(x) = \cosh(x)$, $f'(x) = \operatorname{senh}(x)$, $f''(x) = \cosh(x)$, $f'''(x) = \operatorname{senh}(x)$, $f^{(4)}(x) = \cosh(x)$

Así:

$$\frac{\cosh(0) \cdot x^0}{0!} + \frac{\operatorname{senh}(0) \cdot x^1}{1!} + \frac{\cosh(0) \cdot x^2}{2!} + \frac{\operatorname{senh}(0) \cdot x^3}{3!} + \frac{\cosh(0) \cdot x^4}{4!} \dots$$

$$\frac{x^0}{0!} + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \quad \frac{x^{2k}}{(2k)!}$$

Se reescribe:

$$\sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

f)

$$\tan^{-1}(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1}$$

Derivando $\tan^{-1}(x)$

$f(x) = \tan^{-1}(x)$, $f'(x) = \frac{1}{x^2+1}$, $f''(x) = -\frac{2x}{(x^2+1)^2}$, $f'''(x) = \frac{2(3x^2+1)}{(x^2+1)^3}$, $f^{(4)}(x) = -\frac{24x(x^2-1)}{(x^2+1)^4} \dots$

Así:

$$\frac{\tan^{-1}(0) \cdot x^0}{0!} + \frac{1}{0^2+1} \cdot \frac{x^1}{1!} - \frac{2(0)}{(0^2+1)^2} \cdot \frac{x^2}{2!} + \frac{2(3 \cdot 0^2+1)}{(0^2+1)^3} \cdot \frac{x^3}{3!} - \frac{24(0)(0^2-1)}{(0^2+1)^4} \cdot \frac{x^4}{4!} \dots$$

$\frac{x^1}{1!} - \frac{x^3}{3} + \frac{x^5}{5} + \dots \quad (-1)^k \cdot \frac{x^{2k+1}}{2k+1}$

Se reescribe:

$\sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k+1}}{2k+1}$

g) $\operatorname{sen}^{-1}(x) = \sum_{k=0}^{\infty} \frac{(2n)!}{(2^n n!)(2n+1)} \cdot x^{2k+1}$

Derivando $\operatorname{sen}^{-1}(x)$

$f(x) = \operatorname{sen}^{-1}(x)$, $f'(x) = \frac{1}{\sqrt{1-x^2}}$, $f''(x) = \frac{x}{(1-x^2)^{3/2}}$, $f'''(x) = \frac{2x^2+1}{(1-x^2)^{5/2}}$, $f^{(4)}(x) = \frac{6x^3+9x}{(1-x^2)^{7/2}}$

Así:

$\frac{\operatorname{sen}^{-1}(0) \cdot x^0}{0!} + \frac{1}{\sqrt{1-0^2}} \cdot \frac{x^1}{1!} + \frac{0}{(1-0^2)^{3/2}} \cdot \frac{x^2}{2!} + \frac{2(0^2)+1}{(1-0^2)^{5/2}} \cdot \frac{x^3}{3!} + \frac{6(0^3)+9(0)}{(1-0^2)^{7/2}} \cdot \frac{x^4}{4!} + \dots$

$\frac{x^1}{1!} + \frac{x^3}{3!} + \frac{9x^5}{5!} + \dots$ No se encuentra relación para la suma

h) $\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} \cdot x^{k+1}$

Derivando $\ln(1+x)$

$f(x) = \ln(1+x)$, $f'(x) = \frac{1}{x+1}$, $f''(x) = -\frac{1}{(x+1)^2}$, $f'''(x) = \frac{2}{(x+1)^3}$, $f^{(4)}(x) = -\frac{6}{(x+1)^4}$

Así:

$\ln(1+0) \cdot \frac{x^0}{0!} + \frac{1}{0+1} \cdot \frac{x^1}{1!} - \frac{1}{(0+1)^2} \cdot \frac{x^2}{2!} + \frac{2}{(0+1)^3} \cdot \frac{x^3}{3!} - \frac{6}{(0+1)^4} \cdot \frac{x^4}{4!} + \dots$

$\frac{x^1}{1} - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad (-1)^k \cdot \frac{x^{k+1}}{k+1}$

Se reescribe:

$\sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{k+1}}{k+1}$

i) $\frac{1}{1+x} = \sum_{k=1}^{\infty} (-1)^k \cdot x^k$

Derivando $\frac{1}{1+x}$

$f(x) = \frac{1}{1+x}$, $f'(x) = -\frac{1}{(x+1)^2}$, $f''(x) = \frac{2}{(x+1)^3}$, $f'''(x) = -\frac{6}{(x+1)^4}$, $f^{(4)}(x) = \frac{24}{(x+1)^5}$

Así:

$\frac{1}{1+0} \cdot \frac{x^0}{0!} - \frac{1}{(0+1)^2} \cdot \frac{x^1}{1!} + \frac{2}{(0+1)^3} \cdot \frac{x^2}{2!} - \frac{6}{(0+1)^4} \cdot \frac{x^3}{3!} + \frac{24}{(0+1)^5} \cdot \frac{x^4}{4!}$

$x^0 - x^1 + x^2 - x^3 + x^4 + \dots \quad (-1)^k \cdot x^k$

Se reescribe:

$\sum_{k=0}^{\infty} (-1)^k \cdot x^k$

2.- Comprobar que

a) $e^{x^2} = \sum_{k=0}^{\infty} \frac{x^{2k}}{k!}$ en $(-\infty, \infty)$

A partir de: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

Sea $u = x^2$

$1 + \frac{1}{1!} \cdot u + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} + \dots$

$1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \dots \frac{x^{2k}}{k!}$

Se reescribe:

$\sum_{k=0}^{\infty} \frac{x^{2k}}{k!}$ como $u = x^2$, su intervalo es en $(-\infty, \infty)$

b) $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$ en $(-1, 1)$

A partir de: $\frac{1}{1+x} = \sum_{k=1}^{\infty} (-1)^k \cdot x^k$

Sea $u = -x$

$u^0 - u^1 + u^2 - u^3 + u^4 + \dots$

$1 + x + x^2 + x^3 + x^4 + \dots \quad x^k$

Se reescribe:

$\sum_{k=0}^{\infty} x^k$ en $(-1, 1)$

c) $\frac{1}{x} = \sum_{k=0}^{\infty} (-1)^k (x-1)^k$ en $(0, 2)$

A partir de $\frac{1}{1+x} = \sum_{k=1}^{\infty} (-1)^k \cdot x^k$

Sea $u = x - 1$

$u^0 - u^1 + u^2 - u^3 + u^4 + \dots$

$1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 + \dots (-1)^k \cdot (x-1)^k$

Se reescribe:

$\sum_{k=0}^{\infty} (-1)^k \cdot (x-1)^k$ en $(0, 2)$

d) $\ln(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} \cdot (x-1)^{k+1}$ en $(0, 2]$

A partir de: $\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} (x)^{k+1}$

Sea $u = x-1$

$\frac{u^1}{1} - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \ldots$

$\frac{(x-1)}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}, \ldots \quad (-1)^k \cdot \frac{(x-1)^{k+1}}{(k+1)}$

Se reescribe:

$\sum_{k=0}^{\infty} \frac{(-1)^k \cdot (x-1)^{k+1}}{k+1}$

e) $\ln(1-x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} \cdot x^{k+1}$

A partir de: $\ln(1+x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} \cdot (x)^{k+1}$

Sea $u = -x$

$\frac{u^1}{1} - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \ldots$

$-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \ldots - \frac{x^{k+1}}{k+1}$

Se reescribe

$\sum_{k=0}^{\infty} - \frac{x^{k+1}}{k+1}$

Fecha de Entregas: 17 de junio del 2019

Tarea 19: Series de Taylor y Maclaurin (Segunda Parte)

1- Por medio de series aproximar

a) $\lim_{x \to 0} \frac{e^x - (1-x)}{x^2}$

$$\frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots\right) - (1-x)}{x^2}$$

$$\frac{2x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots}{x^2}$$

$$\lim_{x \to 0} \frac{x^2\left(\frac{2}{x} + \frac{1}{2!} + \frac{x}{3!} + \frac{x^2}{4!} + \dots\right)}{x^2}$$

evaluando el límite tiende a infinito

b) $\lim_{x \to 0} \frac{x^3}{x - \operatorname{sen}(x)}$

$$\lim_{x \to 0} \frac{x^3}{x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right)}$$

$$\lim_{x \to 0} \frac{x^3}{x^3\left(-\frac{1}{3!} + \frac{x^2}{5!} - \frac{x^4}{7!} + \dots\right)}$$

evaluando
-6

c) $\lim_{t \to 0} \frac{1 + t - e^t}{1 - \cos(t)}$

$$\lim_{t \to 0} \frac{1 + t - \left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \dots\right)}{1 - \left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \frac{t^8}{8!} + \dots\right)}$$

$$\lim_{t \to 0} \frac{-\frac{t^2}{2!} - \frac{t^3}{3!} - \frac{t^4}{4!} + \dots}{\frac{t^2}{2!} - \frac{t^4}{4!} + \frac{t^6}{6!} - \frac{t^8}{8!} + \dots} = \lim_{t \to 0} \frac{t^2\left(-\frac{1}{2!} - \frac{t}{3!} - \frac{t^2}{4!} + \dots\right)}{t^2\left(\frac{1}{2!} - \frac{t^2}{4!} + \frac{t^4}{6!} - \frac{t^6}{8!} + \dots\right)}$$

evaluando:
$$\frac{-\frac{1}{2!}}{\frac{1}{2!}} = -1$$

-1

d) $\lim_{y \to 0} \dfrac{y - \ln(1+y)}{y^2}$

$\lim_{y \to 0} \dfrac{y - \left(y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + (\cdots)\right)}{y^2}$

$\lim_{y \to 0} \dfrac{y^2\left(\frac{1}{2} - \frac{y}{3} + \frac{y^2}{4} + \cdots\right)}{y^2}$

evaluando:

$\frac{1}{2}$

e) $\lim_{\theta \to 0} \dfrac{\operatorname{sen}\theta - \theta + \frac{\theta^3}{6}}{\theta^5}$

$\lim_{\theta \to 0} \dfrac{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right) - \theta + \frac{\theta^3}{6}}{\theta^5}$

$\lim_{\theta \to 0} \dfrac{\theta^5\left(\frac{1}{120} - \frac{\theta^2}{7!} + \cdots\right)}{\theta^5}$

evaluando

$\frac{1}{120}$

f) $\lim_{x \to 0} \dfrac{1 - \cos(x)}{1 + x - e^x}$

$\lim_{x \to 0} \dfrac{1 - \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right)}{1 + x - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)}$

$\lim_{x \to 0} \dfrac{x^2\left(\frac{1}{2!} - \frac{x^2}{4!} + \frac{x^4}{6!} + \cdots\right)}{x^2\left(-\frac{1}{2!} - \frac{x}{3!} - \frac{x^2}{4!} + \cdots\right)}$

evaluando

$\dfrac{\frac{1}{2}}{-\frac{1}{2}} = -1$

-1

g) $\lim\limits_{r \to 2} \dfrac{r^2-4}{\ln(r-1)}$

Se debe conocer una nueva serie:

$$\sum_{k=0}^{\infty} \frac{(-1)^k (x-2)^{k+1}}{k+1} = \ln(x-1)$$

$$\lim_{r \to 2} \frac{(r-2)(r+2)}{\left(r-2-\frac{(r-2)^2}{2}+\frac{(r-2)^3}{3}+\dots\right)}$$

$$\frac{r+2}{1-\frac{r-2}{2}+\frac{(r-2)^2}{3}-\frac{(r-2)^3}{4}+\frac{(r-2)^4}{5}+\dots}$$

evaluando

4

2- Estimar los valores de las siguientes integrales con un error de magnitud

a) Menor a 10^{-3}

i) $\displaystyle\int_0^{1/5} \operatorname{sen}(t^2)\, dt$

Se debe conocer una nueva serie:

$$\sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{4k+2}}{(2k+1)!}$$

$$\int_0^{1/5} \sum_{k=0}^{\infty} \frac{(-1)^k \cdot t^{4k+2}}{(2k+1)!}\, dt$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \int_0^{1/5} t^{4k+2}\, dt$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \cdot \frac{t^{4k+3}}{4k+3} \Bigg|_0^{1/5}$$

Así:

$$\frac{1}{375} - \frac{1}{3281250} + \dots \approx 0.002664$$

ii) $\displaystyle\int_0^{1/5} \frac{e^{-x}-1}{x}\, dx$

Se debe conocer una nueva serie:

$$\sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^k}{k!}$$

$$\int_0^{1/5} \sum_{k=1}^{\infty} \frac{(-1)^k \cdot x^{k-1}}{k!}\, dx$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k!} \cdot \frac{x^k}{k} \Bigg|_0^{1/5}$$

Así:

$$-\frac{1}{5} + \frac{1}{100} - \frac{1}{2250} + \dots \approx -\frac{857}{4500} = -0.1904$$

iii) $\int_0^{\frac{1}{10}} \frac{dx}{\sqrt{1+x^4}}$

Por la relación de $\operatorname{sen}^{-1}(\theta)$ incalculable, se procederá sin uso de series

Las pruebas no arrojan una antiderivada

iv) $\int_0^{1/4} \sqrt[3]{1+t^2}\, dt$

Las pruebas no arrojan una antiderivada

b) Menor a 10^{-8}

i) $\int_0^{\frac{1}{10}} \frac{\operatorname{sen}(r)}{r}\, dr$

$\int_0^{\frac{1}{10}} \sum_{k=1}^{\infty} \frac{(-1)^k \cdot r^{2k+1}}{(2k+1)!}\, dr$

$\sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+1)!} \cdot \frac{r^{2k+2}}{2k+2} \Big|_0^{\frac{1}{10}}$

Así

$\approx -\frac{1}{240000}$

ii) $\int_0^{\frac{1}{5}} e^{-y^2}\, dy$

Se debe conocer:

$\sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^{2k}}{k!}$

$\int_0^{\frac{1}{5}} \sum_{k=0}^{\infty} \frac{(-1)^k \cdot y^{2k}}{k!}\, dy$

$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \cdot \frac{y^{2k+1}}{2k+1} \Big|_0^{\frac{1}{5}}$

Así:

$\frac{1}{5} - \frac{1}{375} + \frac{1}{31250} - \frac{1}{3281250} + 2.3703 \times 10^{-4}$

≈ 0.1973650309

iii) $\int_0^1 \frac{1-\cos(t)}{t^2}\, dt$

$\int_0^1 \sum_{k=1}^{\infty} \frac{(-1)^k \cdot t^{2k-2}\, dt}{(2k)!} = \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k)!} \cdot \frac{t^{2k-1}}{2k-1} \Big|_0^1$

Así

$-\frac{1}{2} + \frac{1}{72} - \frac{1}{3600} + \frac{1}{282240} - \frac{1}{32659200}$

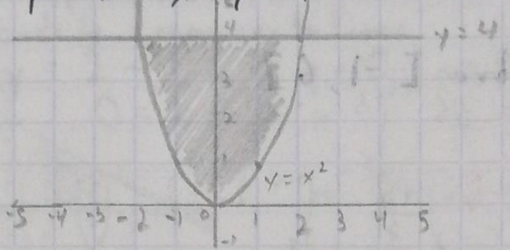
≈ -0.4863853764

Fecha de Entrega: 17 de junio del 2019

Tarea 20: Área de una región entre dos curvas

1- Determinar el área de la región acotada por la gráfica de las funciones dadas

a) $y = x^2$, $y = 4$

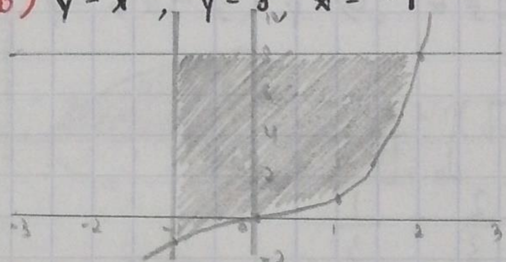


$$\int_{-2}^{2} 4\,dx = 4x\Big|_{-2}^{2} = 16$$

$$\int_{-2}^{2} x^2\,dx = \frac{x^3}{3}\Big|_{-2}^{2} = \frac{16}{3}$$

$$\therefore A = 16 - \frac{16}{3} = \frac{32}{3}$$

b) $y = x^3$, $y = 8$, $x = -1$

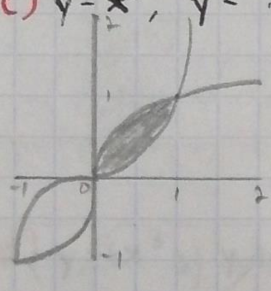


$$\int_{-1}^{2} 8\,dx = 24$$

$$\int_{0}^{2} x^3\,dx = 4, \quad \int_{-1}^{0} x^3\,dx = -\frac{1}{4}$$

$$\therefore A = 24 - \left(4 - \frac{1}{4}\right) = 20.25$$

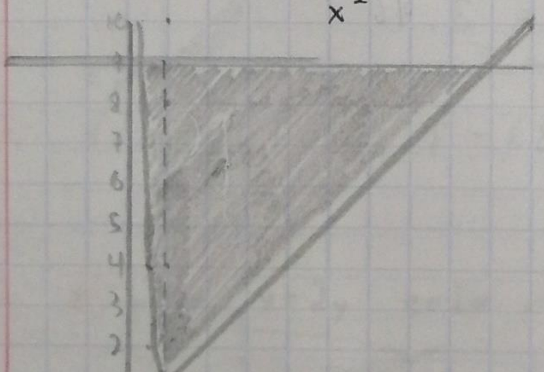
c) $y = x^3$, $y = \sqrt[3]{x}$ en el 1^er cuadrante



$$\int_{0}^{1} \sqrt[3]{x}\,dx = \frac{3}{4}, \quad \int_{0}^{1} x^3\,dx = \frac{1}{4}$$

$$\therefore A = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

d) $y = x$, $y = \frac{1}{x^2}$, $y = 9$ en el 1^er cuadrante



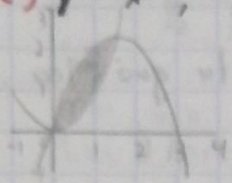
$$\int_{1/3}^{9} 9\,dx = 78$$

$$\int_{1/3}^{1} \frac{1}{x^2}\,dx = 2$$

$$\int_{1}^{9} x\,dx = 40$$

$$\therefore A = 78 - 2 - 40 = 36$$

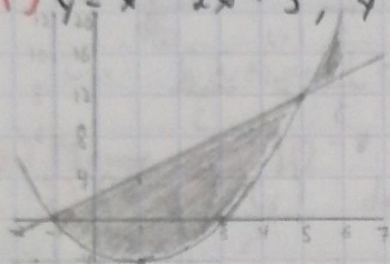
e) $y = x^2, \quad y = 3x - x^2$



$\int_0^{1.5} 3x - x^2 = 2.25, \quad \int_0^{1.5} x^2 = 1.125$

$\therefore A = 2.25 - 1.125 = 1.125$

f) $y = x^2 - 2x - 3, \quad y = 2x + 2$ sobre $[-1, 6]$

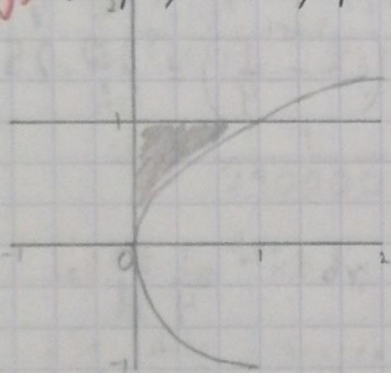


$\int_{-1}^{5} 2x + 2 = 36, \quad \int_{1}^{5} x^2 - 2x - 3 = 0$

$\int_{5}^{6} x^2 - 2x - 3 = \frac{49}{3}, \quad \int_{5}^{6} 2x + 2 = 13$

$\therefore A = 36 + \left(\frac{49}{3} - 13\right) = \frac{118}{3}$

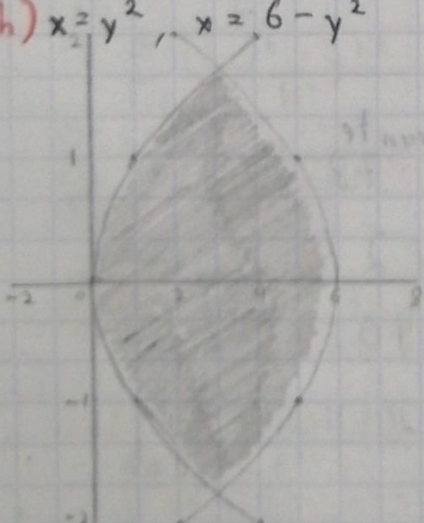
g) $x = y^2, \quad x = 0, \quad y = 1$



$\int_0^1 dx = 1, \quad \int_0^1 \sqrt{x}\, dx = \frac{2}{3}$

$\therefore A = 1 - \frac{2}{3} = \frac{1}{3}$

h) $x = y^2, \quad x = 6 - y^2$



Se integrará en función de y

$\int_{-\sqrt{3}}^{\sqrt{3}} 6 - y^2\, dy = 17.32$

$\int_{-\sqrt{3}}^{\sqrt{3}} y^2\, dy = 3.46$

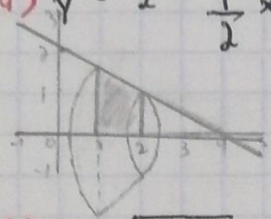
$\therefore A = 17.32 - 3.46 = 13.856$

Fecha de Entrega: 17 de Junio del 2019

Tarea 21: Volumen de un Sólido de Revolución

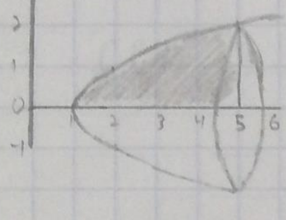
1- Determinar el volumen del sólido obtenido al hacer girar la región delimitada por las curvas dadas alrededor de la recta especificada. Graficar la región, el sólido y un disco o arandela representativos

a) $y = 2 - \frac{1}{2}x$, $y = 0$, $x = 1$, $x = 2$ sobre el eje x



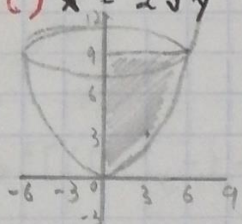
$$\pi \cdot \int_1^2 \left(2 - \frac{1}{2}x\right)^2 dx = \frac{19\pi}{12} \ u^3$$

b) $y = \sqrt{x-1}$, $y = 0$, $x = 5$ sobre el eje x



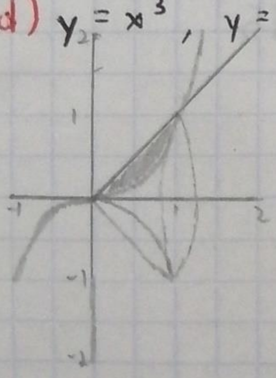
$$\pi \cdot \int_1^5 \left(\sqrt{x-1}\right)^2 dx = 8\pi \ u^3$$

c) $x = 2\sqrt{y}$ $x = 0$ $y = 9$ sobre el eje y



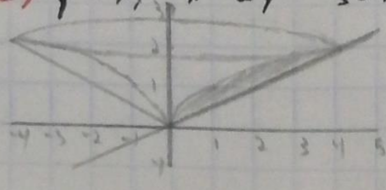
$$\pi \int_0^9 \left(2\sqrt{y}\right)^2 dy = 162\pi \ u^3$$

d) $y = x^3$, $y = x$, $x \geq 0$, sobre el eje x



$$\pi \int_0^1 \left(x - x^3\right)^2 dx = \frac{8\pi}{105}$$

e) $y^2 = x$, $x = 2y$ sobre el eje y

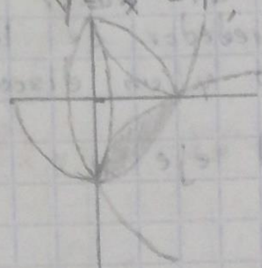


$$\pi \int_0^2 \left(2y - y^2\right)^2 dy = \frac{16\pi}{15}$$

f) $y = x^2$, $x = y^2$ sobre $y = 1$

Para girar sobre x se trasladará

$y = x^2 - 1$, $x = (y+1)^2$

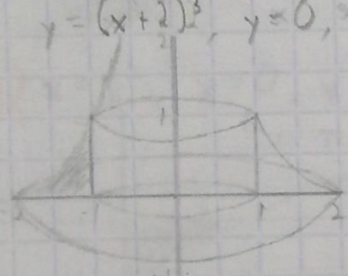


$$\pi \int_0^1 [(x^2 - 1) - (\sqrt{x} - 1)]^2 \, dx = 0.40319$$

g) $y = x^3$, $y = 0$, $x = 1$ sobre $x = 2$

Para girar sobre y se trasladará

$y = (x+2)^3$, $y = 0$, $x = -1$

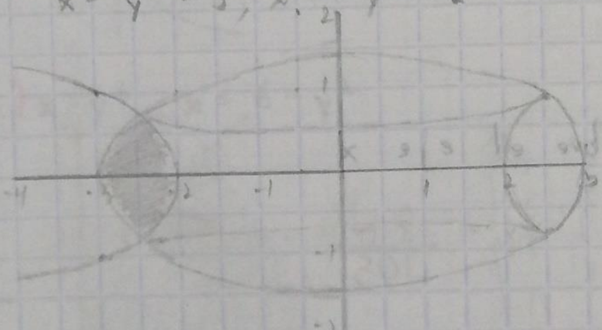


$$\pi \int_0^1 [(\sqrt[3]{y} - 2) - 1]^2 \, dy = 16.022$$

h) $x = y^2$, $x = 1 - y^2$ sobre $x = 3$

Para girar sobre y se trasladará

$x = y^2 - 3$, $x = -y^2 - 2$



$$\pi \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} [(y^2 - 3) - (-y^2 - 2)]^2 \, dx$$

$$= 2.3695$$